## Linear Algebra

### 2000 IIT Kharagpur

1.1 The rank of matrix \( A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix} \) is

(A) 0  
(B) 1  
(C) 2  
(D) 3

1.2 For a singular matrix

(A) at least one Eigen value would be at the origin.

(B) all Eigen values would be at the origin.

(C) no Eigen value would be at the origin.

(D) none.

### 2001 IIT Kanpur

1.3 The necessary condition to diagonalise a matrix is that

(A) its all Eigen values should be distinct.

(B) its Eigen vectors should be independent.

(C) its Eigen values should be real.

(D) the matrix is non-singular.

### 2005 IIT Bombay

1.5 Identify which one of the following is an Eigen vector of the matrix \( A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix} \).

(A) \( \begin{bmatrix} -1 \\ 1 \end{bmatrix} \)  
(B) \( \begin{bmatrix} 3 \\ -1 \end{bmatrix} \)

(C) \( \begin{bmatrix} 1 \\ -1 \end{bmatrix} \)  
(D) \( \begin{bmatrix} -2 \\ 1 \end{bmatrix} \)

1.6 Let \( A \) be a \( 3 \times 3 \) matrix with rank 2. Then \( AX = 0 \) has

(A) only the trivial solution \( X = 0 \).

(B) one independent solution.

(C) two independent solutions.

(D) three independent solutions.

### 2006 IIT Kharagpur

1.7 For a given \( 2 \times 2 \) matrix \( A \), it is observed that \( A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = -2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \). Then matrix \( A \) is

(A) \( \begin{bmatrix} 2 & 1 \\ -1 & -2 \end{bmatrix} \)

Coefficients, has a non-trivial solution when \( A \) is

(A) non-singular  
(B) singular  
(C) symmetric  
(D) hermitian
1.11 Let \( A = [a_{ij}] \), \( 1 \leq i, j \leq n \), with \( n \geq 3 \) and \( a_{ij} = i \cdot j \). Then the rank of \( A \) is

(A) 0 (B) 1 (C) \( n - 1 \) (D) \( n \)

2009 IIT Roorkee

1.12 Let \( P \neq 0 \) be a \( 3 \times 3 \) real matrix. There exist linearly independent vectors \( x \) and \( y \) such that \( Px = 0 \) and \( Py = 0 \). The dimension of the range space of \( P \) is

(A) 0 (B) 1 (C) 2 (D) 3

1.13 The Eigen values of a \( (2 \times 2) \) matrix \( X \) are \( -2 \) and \( -3 \). The Eigen values of the matrix \( (X + I)^{-1} (X + 5I) \) are

(A) \(-3, -4\) (B) \(-1, -2\) (C) \(-1, -3\) (D) \(-2, -4\)

1.14 The matrix \( P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \) rotates a vector about the axis \( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \) by an angle of

(A) \( 30^0 \) (B) \( 60^0 \) (C) \( 90^0 \) (D) \( 120^0 \)

2010 IIT Guwahati

1.15 A real \( n \times n \) matrix \( A = [a_{ij}] \) is defined as follows:

\[ a_{ij} = i \quad \text{if} \quad i = j \]

\[ = 0 \quad \text{otherwise} \]

The summation of all \( n \) Eigen values of \( A \) is

(A) \( \frac{n(n+1)}{2} \) (B) \( \frac{n(n-1)}{2} \) (C) \( \frac{n(n+1)(2n+1)}{6} \) (D) \( n^2 \)
1.16 \( X \) and \( Y \) are non-zero square matrices of size \( n \times n \). If \( XY = 0 \), then

(A) \( |X| = 0 \) and \( |Y| \neq 0 \)
(B) \( |X| \neq 0 \) and \( |Y| = 0 \)
(C) \( |X| = 0 \) and \( |Y| = 0 \)
(D) \( |X| \neq 0 \) and \( |Y| \neq 0 \)

2011 IIT Madras

1.17 The matrix has \( M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \). Eigen values \(-3, -3, 5\). An Eigen vector corresponding to the Eigen value 5 is \( \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}^T \). One of the Eigen vectors of the matrix \( M^3 \) is

(A) \( \begin{bmatrix} 1 \\ 8 \\ -1 \end{bmatrix}^T \)
(B) \( \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}^T \)
(C) \( \begin{bmatrix} 1 \\ \sqrt{2} \\ -1 \end{bmatrix}^T \)
(D) \( \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}^T \)

2012 IIT Delhi

1.18 Given that \( A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \) and \( I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \), the value of \( A^3 \) is

(A) \( 15A + 12I \)
(B) \( 19A + 30I \)
(C) \( 17A + 15I \)
(D) \( 17A + 21I \)

2013 IIT Bombay

1.19 The dimension of the null space of the matrix \( \begin{bmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \) is

(A) 0
(B) 1
(C) 2
(D) 3

2014 IIT Kharagpur

1.20 One pair of Eigen vectors corresponding to the two Eigen values of the matrix \( \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \) is

(A) \( \begin{bmatrix} 1 \\ j \end{bmatrix} \), \( \begin{bmatrix} j \\ -1 \end{bmatrix} \)
(B) \( \begin{bmatrix} 0 \\ -1 \end{bmatrix} \), \( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \)
(C) \( \begin{bmatrix} 1 \\ 0 \end{bmatrix} \), \( \begin{bmatrix} j \\ 1 \end{bmatrix} \)
(D) \( \begin{bmatrix} 1 \\ j \end{bmatrix} \), \( \begin{bmatrix} j \end{bmatrix} \)

2015 IIT Kanpur

1.21 For the matrix \( A \) satisfying the equation given below, the Eigen values are

\[ \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 & 6 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \]

(A) \( (1, -j, j) \)
(B) \( (1, 1, 0) \)
(C) \( (1, 1, -1) \)
(D) \( (1, 0, 0) \)

2016 IISc Bangalore

1.22 Let \( A \) be an \( n \times n \) matrix with rank \( r \), \( (0 < r < n) \). Then \( AX = 0 \) has \( p \) independent solutions, where \( p \) is

(A) \( r \)
(B) \( n \)
(C) \( n - r \)
(D) \( n + r \)

1.23 A straight line of the form \( y = mx + c \) passes through the origin and the point \((x, y) = (2, 6)\). The value of \( m \) is _______.

1.24 Consider the matrix \( A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix} \) whose Eigen values are 1, -1 and 3. The trace of \( A^3 - 3A^2 \) is _______.
1.25 The eigenvalues of the matrix

\[ A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix} \]

are

(A) \(-1, 5, 6\)  
(B) \(1, -5 \pm j6\)  
(C) \(1, 5 \pm j6\)  
(D) \(1, 5, 5\)

1.26 If \(V\) is a non-zero vector of dimension \(3 \times 1\), then the matrix \(A = VV^T\) has rank = ______.

1.27 The figure shows a shape \(ABC\) and its mirror image \(A_1B_1C_1\) across the horizontal axis (X-axis). The coordinate transformation matrix that maps \(ABC\) to \(A_1B_1C_1\) is

(A) \(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\)  
(B) \(\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\)  
(C) \(\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\)  
(D) \(\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\)

1.28 Let \(N\) be a \(3 \times 3\) matrix with real number entries. The matrix is such that \(N^2 = 0\). The eigenvalues of \(N\) are

(A) \(0, 0, 0\)  
(B) \(0, 0, 1\)  
(C) \(0, 1, 1\)  
(D) \(1, 1, 1\)

1.29 Consider the following system of linear equations:

\[ \begin{align*}
3x + 2ky &= -2 \\
kx + 6y &= 2
\end{align*} \]

Here \(x\) and \(y\) are the unknowns and \(k\) is a real constant. The value of \(k\) for which there are infinite number of solutions is,

(A) \(3\)  
(B) \(1\)  
(C) \(-3\)  
(D) \(-6\)
4.7

Answers

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Explanations

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Given: \[ A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 6 & 8 \end{bmatrix} \]

Method 1

Changing the matrix into echelon form by elementary transformations,

\[ R_2 \rightarrow R_2 - 3R_1 \]

\[ A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 4 & 6 & 8 \end{bmatrix} \]

\[ R_3 \rightarrow R_3 - 4R_1 \]

\[ A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & -2 & -4 \end{bmatrix} \]

\[ R_3 \rightarrow R_3 - R_2 \]

\[ A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \]

There exists two non-zero rows in echelon form. Hence, rank of the matrix

\[ \rho(A) = 2 \]

Hence, the correct option is (C).

Method 2

The rank of a matrix is defined as the number of linearly independent rows/columns whichever is minimum, present in the matrix.

In the given matrix, there exist a linear relationship as given below,

\[ R_3 = R_1 + R_2 \]

So, there are only two linearly independent rows \( R_1 \) and \( R_3 \).

\[ \rho(A) = 2 \]

Hence, the correct option is (C).

Key Point

The elementary transformations are done in order to convert matrix to its Echelon form.

The number of non-zero rows remaining after elementary transformations gives the rank of that matrix.

A matrix is in its Echelon form if:

(i) Leading non-zero elements of a row are behind the leading non-zero elements in its previous row.

(ii) All the zero rows should be below all the non-zero rows.

This method is also known as Gauss-elimination method.
A singular matrix is a square matrix whose determinant is zero i.e. $|A| = 0$.

By the property of square matrix, the determinant is equal to the product of its Eigen values.

$$|A|_{\text{det}} = \lambda_1 \times \lambda_2 \times \lambda_3 \ldots \lambda_n$$

Therefore, for singular matrix, atleast one of the Eigen values should be zero.

Hence, the correct option is (A).

**Key Point**

If the determinant of a matrix is zero, then it is called a **Singular matrix**. For singular matrix, $|A| = 0$

**Necessary condition**:

By the concept of diagonalization, every square matrix $A$ of order $n$ is diagonalizable if and only if it has $n$ linearly independent Eigen vectors.

Also, for a matrix to be diagonalizable, all its Eigen values should be distinct.

Hence, the correct options are (A) & (B).

**Key Point**

**Concept of diagonalization**:

Any square matrix with all its distinct Eigen values can be represented as,

$$A = [M] [D] [M^{-1}]$$

where,

$D =$ Diagonal matrix whose diagonal elements are Eigen values of $A$.

$M =$ Non-singular matrix whose columns are respective Eigen vectors of $A$ i.e. the Eigen vectors should be independent.

$M$ is also referred to as **Modal matrix**.

**Given**:

A system of equations is given by,

$$AX = 0$$

This represents homogeneous equation.

For non-trivial solution, $|A| = 0$

which also represents condition for singular matrix.

Hence, the correct option is (B).

**Key Point**

**For a homogenous equation**:

$$AX = 0$$

- Unique solution $|A| \neq 0$
  - Rank $(A) = n$

- Infinitely many solutions $|A| = 0$
  - Rank $(A) < n$
  - $[(n - r) \text{ linearly independent non-trivial solution}]$

**Note**:

A homogenous equation is always consistent (at least one solution exists).

**Given**:

$$A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix}$$

**Method 1**

The characteristics equation is given by,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1 - \lambda & 0 \\ -1 & -2 - \lambda \end{vmatrix} = 0$$

$$1 - \lambda - (-2 - \lambda) = 0$$

$$-2 - \lambda + 2\lambda + \lambda^2 = 0$$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_1 + 2)(\lambda - 1) = 0$$

$$\lambda = 1, \lambda = -2$$

The roots of characteristics equations of a matrix give its Eigen values.

For any Eigen vector $[X]$, for a matrix $[A]$ corresponding to Eigen value $\lambda$, the following equation satisfies,
For \( \lambda = 1 \),
\[
\begin{bmatrix}
0 & 0 \\
-1 & -3
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
On solving,
\[-x - 3y = 0 \Rightarrow -x = 3y\]
\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = k
\begin{bmatrix}
3 \\
-1
\end{bmatrix}
\]
For \( k = 1 \),
\[
\begin{bmatrix}
x \\
y
\end{bmatrix} =
\begin{bmatrix}
3 \\
-1
\end{bmatrix}
\]
Hence, the correct option is (B).

**Method 2**

\[
A =
\begin{bmatrix}
1 & 0 \\
-1 & -2
\end{bmatrix}
\]
For any Eigen vector \([X]\), for a matrix \([A]\) corresponding to Eigen value \(\lambda\), the following equation satisfies,
\[
[A - \lambda I][X] = 0
\]
\[
AX = \lambda X \quad \text{(i)}
\]
**Calculation of Eigen values :**
The given matrix has all the elements above main diagonal equal to zero. So, matrix \(A\) is a lower triangular matrix.
So, the Eigen values are the diagonal elements.
\(\lambda_1 = 1, \lambda_2 = -2\)

**From option (A) :**
\[
X =
\begin{bmatrix}
-1 \\
1
\end{bmatrix}
\]
and \(\lambda = 1\)
From equation (i),
\[
\begin{bmatrix}
1 & 0 \\
-1 & -2
\end{bmatrix}
\begin{bmatrix}
3 \\
-1
\end{bmatrix} = 1 \times \begin{bmatrix}
3 \\
-1
\end{bmatrix}
\]
Hence, option (A) is incorrect.

**From option (B) :**
\[
X =
\begin{bmatrix}
3 \\
-1
\end{bmatrix}
\]
and \(\lambda = 1\)
From equation (i),
\[
\begin{bmatrix}
1 & 0 \\
-1 & -2
\end{bmatrix}
\begin{bmatrix}
3 \\
-1
\end{bmatrix} = 1 \times \begin{bmatrix}
3 \\
-1
\end{bmatrix}
\]
Hence, the correct option is (B).

**Key Point**
For any upper or lower triangular matrix,
(i) the elements of main diagonal are the Eigen values of the matrix.
(ii) the product of diagonal elements gives determinant of the matrix.

**1.6 (B)**
**Given :** \(A\) is a \(3 \times 3\) matrix.
\[\rho(A) = r = 2\]
Order of matrix \((n) = 3\)
The equation \(AX = 0\) represents a homogeneous equation.
The number of linearly independent solutions is given by,
\[n - r = 3 - 2 = 1\]
Hence, the correct option is (B).

**1.7 (C)**
**Given :** \(A\) is a \(2 \times 2\) matrix such that,
\[
A
\begin{bmatrix}
1 \\
-1
\end{bmatrix} =
\begin{bmatrix}
1 \\
-1
\end{bmatrix} \quad \text{(i)}
\]
\[
A
\begin{bmatrix}
1 \\
-2
\end{bmatrix} =
\begin{bmatrix}
1 \\
-2
\end{bmatrix} \quad \text{(ii)}
\]
For any Eigen vector \([X]\), for a matrix \([A]\) corresponding to Eigen value \(\lambda\), the following equation satisfies,
$[A - \lambda I][X] = 0$
$AX = \lambda X$

On comparing above equation with equations (i) and (ii),

$\lambda_1 = -1, \quad X_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda_2 = -2, \quad X_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

Modal matrix for a square matrix is given by,

$[M] = [X_1 \quad X_2]$

$[M] = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix}$

From the concept of diagonalization, the matrix $A$ is given by,

$[A] = [M] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [M]^{-1}$ ...(iii)

Inverse of $M$ is given by,

$M^{-1} = \frac{1}{-1} \begin{bmatrix} -2 & -1 \\ 1 & 1 \end{bmatrix}$

$M^{-1} = \begin{bmatrix} 2 & 1 \\ -1 & -1 \end{bmatrix}$

From equation (iii),

$[A] = \begin{bmatrix} 1 & 1 \\ -1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \end{bmatrix}$

Hence, the correct option is (C).

By elementary transformation,

$R_3 \rightarrow R_3 - R_1$

$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_2$

$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

The above form is echelon form and there exist four non-zero rows.

So, $\rho(A) = 4$

Hence, the correct option is (D).

1.8 (D)

Given : $AX = B$

This represents a non-homogenous equation.

Augmented matrix is given by,

$[A : B] = \begin{bmatrix} 1 & 0 & 1 & 0 : 0 \\ 0 & 1 & 0 & 1 : 0 \\ 1 & 1 & 0 & 0 : 0 \\ 0 & 0 & 0 & 1 : 1 \end{bmatrix}$

By elementary transformations,

$R_3 \rightarrow R_3 - R_1$

$[A : B] = \begin{bmatrix} 1 & 0 & 1 & 0 : 0 \\ 0 & 1 & 0 & 1 : 0 \\ 0 & 1 & 1 & 0 : 0 \\ 0 & 0 & 0 & 1 : 1 \end{bmatrix}$

$R_3 \rightarrow R_3 - R_2$

$[A : B] = \begin{bmatrix} 1 & 0 & 1 & 0 : 0 \\ 0 & 1 & 0 & 1 : 0 \\ 0 & 0 & 1 & -1 : 0 \\ 0 & 0 & 0 & 1 : 1 \end{bmatrix}$
There exists four non-zero rows.  
So, $\rho(A) = 4$

$\rho(A : B) = 4$

and number of variables $n = 4$

$\rho(A) = \rho(A : B) = n = 4$

This shows, the given non-homogenous equation has a unique solution.

Hence, the correct option is (B).

**Key Point**

For a non-homogenous equation $AX = B$ :

(i) If rank $[A : B] \neq \text{rank}[A]$,

   $\Rightarrow$ No solution exists.

(ii) If rank $[A : B] = \text{rank}[A] = \text{Number of variables}$,

   $\Rightarrow$ Unique solution exists.

(iii) If rank $[A : B] = \text{rank}[A] < \text{Number of variables}$,

   $\Rightarrow$ Infinite numbers of solutions exist.

**Given :** $A$ is an $n \times n$ real matrix and $A^2 = I$.

Determinant of $A^2$ is given by,

$|A^2| = |I| = 1$

$|A| = \pm 1$

$|A| \neq 0$

[Condition for unique solution]

Therefore, $Ax = y$ is consistent and has unique solution given by $x = A^{-1}y$.

Hence, the correct option is (B).

**Given :** A matrix is defined as, $A = [a_{ij}]$

where, $1 \leq i, j \leq n$ and $n \geq 3$ and $a_{ij} = i.j$

For $n = 3 \Rightarrow j \leq 3$,

Let order of matrix be $2 \times 3$.

Then, $[A]_{2\times3} = \begin{bmatrix} 1 \times 1 & 1 \times 2 & 1 \times 3 \\ 2 \times 1 & 2 \times 2 & 2 \times 3 \end{bmatrix}$

$[A]_{2\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$

In the above matrix, there exist a linear relationship as given below,

$R_2 = 2R_1$

Thus, only row $R_1$ is an independent row.

So, $\rho(A) = 1$

For $n = 4 \Rightarrow j \leq 4$,

Now, let order of matrix be $3 \times 4$.

Then, $[A]_{3\times4} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}_{3\times4}$

In the above matrix, there exist a linear relationship as given below,

$R_2 = 2R_1$ and $R_3 = 3R_1$

Thus, only row $R_1$ is an independent row.

So, $\rho(A) = 1$

Therefore, for any order of given matrix, rank of $A$ is always equal to 1.

Hence, the correct option is (B).

**Given :** $P$ is a $3 \times 3$ matrix and $P \neq 0$

So, its rank can be 0, 1, 2, 3.

Since, $P \neq 0$ which means $P$ is not a null matrix. Therefore, rank zero is not possible.

$Px = 0$ and $Py = 0$, $P \neq 0$

Therefore, $x = 0, y = 0$ i.e. they form null space which means $P$ has two nullity.

$\text{Rank} = \text{Dimension} - \text{Nullity}$

$\rho = 3 - 2 = 1$

Hence, the correct option is (B).
1.13 (C)

Given: $X$ is a $2 \times 2$ matrix.

Eigen values are $\lambda_1 = -2$ and $\lambda_2 = -3$.

By Cayley Hamilton theorem, every square matrix satisfies its own characteristic equation.

The characteristic equation is given by,

$$|A - \lambda I| = 0$$

$$AI = \lambda I \implies A = \lambda$$

The above expression shows that the values of $\lambda$ can be put in any expression of the matrix $A$.

For $\lambda_1 = -2$,

Eigen value of matrix $[X + I]^{-1}[X + 5I]$ is given by,

$$[X + I]^{-1}[X + 5I] = (2 + 1)^{-1}(2 + 5) = -3$$

For $\lambda_2 = -3$,

Eigen value of matrix $[X + I]^{-1}[X + 5I]$ is given by,

$$[X + I]^{-1}[X + 5I] = (3 + 1)^{-1}(3 + 5) = -1$$

Thus, Eigen values for matrix $(X + I)^{-1}(X + 5I)$ are $-3$ and $-1$.

Hence, the correct option is (C).

Method 1

This represents the axis $x = y = z$.

The matrix $P$ is a rotation matrix which is the matrix of any even permutation which rotates through $120^0$ about the axis $x = y = z$.

Hence, the correct option is (D).

Method 2

Trace of a three dimensional rotation matrix is given by,

$$\text{Trace} = 1 + 2 \cos \theta$$

where, $\theta$ = Angle by which rotation matrix rotates a vector.

Trace of matrix $[P] = 0$

$$1 + 2 \cos \theta = 0$$

$$\cos \theta = -\frac{1}{2}$$

$$\theta = 120^0$$

Hence, the correct option is (D).

1.15 (A)

Given: A matrix is defined as, $A = [a_{ij}]_{n \times n}$

where, $a_{ij} = \begin{cases} i, & i = j \\ 0, & \text{Otherwise} \end{cases}$
Thus, all the elements except diagonal are zero and diagonal elements are given by,

\[ a_{11} = 1, \ a_{22} = 2, \ a_{33} = 3, \ldots, a_{nn} = n \]

\[ A = \begin{bmatrix}
1 & 0 & 0 & \cdots & 0 \\
0 & 2 & 0 & \cdots & 0 \\
0 & 0 & 3 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & n
\end{bmatrix}_{n \times n} \]

The sum of Eigen values is given by the sum of its main diagonal elements.

\[ = (1 + 2 + 3 + \ldots + n) = \frac{n(n+1)}{2} \]

Hence, the correct option is (A).

**Key Point**

If \( \lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n \) are the Eigen values of matrix \( A \), then

(i) according to the property of Eigen values, the Eigen values of \( A^k \) will be \( \lambda_1^k, \lambda_2^k, \lambda_3^k, \ldots, \lambda_n^k \).

(ii) according to the property of Eigen vectors, \( A \) and \( A^k \) will have same Eigen vectors.
Given: \[ A = \begin{bmatrix} -5 & -3 \\ 2 & 0 \end{bmatrix} \]

The characteristic equation is given by,
\[ |A - \lambda I| = 0 \]
\[ \begin{vmatrix} -5 - \lambda & -3 \\ 2 & -\lambda \end{vmatrix} = 0 \]
\[ 5\lambda + \lambda^2 + 6 = 0 \]

By Cayley Hamilton theorem, every square matrix satisfies its own characteristic equation.
\[ 5A + A^2 + 6I = 0 \]
\[ A^2 = -5A - 6I \] ...(i)

Multiplying by \( A \) on both sides,
\[ A^3 = -5A^2 - 6A \]

From equation (i),
\[ A^3 = -5(-5A - 6I) - 6A \]
\[ A^3 = 19A + 30I \]

Hence, the correct option is (B).

1.20 (A) & (D)

Given: \[ A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \]

The characteristics equation is given by,
\[ |A - \lambda I| = 0 \]
\[ \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = 0 \]
\[ \lambda^2 + 1 = 0 \]
\[ \lambda^2 = -1 \]
\[ \lambda_1 = j \text{ and } \lambda_2 = -j \]

For any Eigen vector \([X] \), for a matrix \([A] \) corresponding to Eigen value \( \lambda \),
\[ [A - \lambda I][X] = 0 \]

\( X \) represents the Eigen vector.

For \( \lambda_1 = j \),
\[ \begin{bmatrix} -j & -1 \\ 1 & -j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]
\[ -jx_1 - x_2 = 0 \]
\[ x_1 - jx_2 = 0 \]
\[ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} j \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -j \\ 1 \end{bmatrix} \]

For \( \lambda_2 = -j \),
\[ \begin{bmatrix} j & -1 \\ 1 & j \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]
\[ jx_1 - x_2 = 0 \]
\[ x_1 + jx_2 = 0 \]
One pair of Eigen vectors corresponding to the two Eigen values can be following:

\[
\begin{bmatrix} j \\ 1 \end{bmatrix}, \begin{bmatrix} j \\ -1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} j \\ 1 \end{bmatrix}, \begin{bmatrix} j \\ -j \end{bmatrix}
\]

or

\[
\begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ 1 \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} 1 \\ -j \end{bmatrix}, \begin{bmatrix} j \\ -j \end{bmatrix}
\]

Hence, the correct options are (A) and (D).

**1.21 (C)**

Given: An equation is as given below,

\[
\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}
\]

It can be clearly seen that \( A \) is a \( 3 \times 3 \) square matrix.

Taking the determinant on both sides,

\[
|A| = 1 \times 2 \times 3 = 1 \times 2 \times 3
\]

By exchanging the rows \( R_2 \) and \( R_3 \) in the R.H.S.

\[
|A| = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \\ 4 & 5 & 6 \end{bmatrix}
\]

By the properties of square matrices, the determinant is equal to the product of its Eigen values.

\[
\lambda_1 \times \lambda_2 \times \lambda_3 = |A| = -1
\]

From option (A):

\[
1 \times (-j) \times j = 1 \]

[Incorrect]

From option (B):

\[
1 \times 1 \times 0 = 0 \]

[Incorrect]

From option (C):

\[
1 \times 1 \times (-1) = -1
\]

Hence, the correct option is (C).

**Key Point**

While performing elementary transformation operations, when two lines (either row or zero) are interchanged \( n \) times then

\[
|A|_{\text{new}} = (-1)^n |A|
\]

**1.22 (C)**

Given: Matrix \( A \) is an \( n \times n \) matrix with rank \( r \).

\[
AX = 0
\]

This is a homogeneous equation.

Order of matrix is \( n \).

Rank of matrix \( \rho(A) = r \).

The number of independent solutions for a homogeneous equations are given by,

\[
p = n - r
\]

Hence, the correct option is (C).

**1.23 3**

Given: Straight line, \( y = mx + c \)

passes through points, \((x_1, y_1) = (0, 0)\)

and \((x_2, y_2) = (2, 6)\)

Slope of straight line is given by,

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

So,

\[
m = \frac{6 - 0}{2 - 0} = \frac{6}{2} = 3
\]

Hence, the value of \( m \) is 3.

**1.24 - 6**

Given: \( A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{bmatrix} \)

Eigen values are \( \lambda_1 = 1, \lambda_2 = -1 \) and \( \lambda_3 = 3 \).
By Cayley Hamilton theorem, every square matrix satisfies its own characteristic equation.

The characteristic equation is given by,

\[ |A - \lambda I| = 0 \]

\[ AI = \lambda I \Rightarrow A = \lambda \]

The above expression shows that the values of \( \lambda \) can be put in any expression of the matrix \( A \).

For \( \lambda_1 = 1 \),

Eigen value of \( A^3 - 3A^2 \) is given by,

\[ A^3 - 3A^2 = 1^3 - 3 \times 1^2 = -2 \]

For \( \lambda_2 = -1 \),

\[ A^3 - 3A^2 = (-1)^3 - 3 \times (-1)^2 = -4 \]

For \( \lambda_3 = 3 \),

\[ A^3 - 3A^2 = 3^3 - 3 \times 3^2 = 0 \]

So, trace of matrix \( (A^3 - 3A^2) \)

= Sum of Eigen values

= \((-2) + (-4) + 0 = -6 \)

Hence, the trace of \( (A^3 - 3A^2) \) is \(-6\).

**Key Point**

**For any square matrix:**

(i) Sum of Eigen values is equal to the trace of the matrix (sum of leading diagonal elements).

(ii) Product of Eigen values is equal to the determinant of the matrix.

---

**Given:**

\[
A = \begin{bmatrix} 1 & -1 & 5 \\ 0 & 5 & 6 \\ 0 & -6 & 5 \end{bmatrix}
\]

The characteristic equation is given by,

\[ |A - \lambda I| = 0 \]

\[
\begin{vmatrix} 1-\lambda & -1 & 5 \\ 0 & 5-\lambda & 6 \\ 0 & -6 & 5-\lambda \end{vmatrix} = 0
\]

\((1-\lambda)((5-\lambda)^2 + 36) = 0\)

\((1-\lambda)(\lambda^2 - 10\lambda + 61) = 0\)

\(\lambda_1 = 1, \lambda_2 = 5 + 6j \) and \( \lambda_3 = 5 - 6j \)

The roots of characteristic equations of a matrix give its Eigen values.

Hence, the correct option is (C).

---

**Given:** \( V \) is a non-zero vector of dimension \( 3 \times 1 \).

Let \( V = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \Rightarrow V^T = [x_1 \ x_2 \ x_3] \)

Rank of a matrix \( [A]_{m \times n} \) (non-square matrix) is given by,

\[ \rho(A) \leq \min(\text{row, column}) \]

\[ \rho(A) \leq \min(m, n) \]

So, \( \rho(V) = 1 \)

The rank of transpose of a matrix is equal to the rank of that matrix itself.

So, \( \rho(V^T) = 1 \)

According to question,

\[ A = VV^T \]

Therefore, rank of matrix \( A \) is given by,

\[ \rho(A) = \min\{\rho(V), \rho(V^T)\} \]

\[ \rho(A) = \min\{1, 1\} = 1 \]

Hence, the matrix \( A = VV^T \) has rank \( 1 \).

---

**Key Point**

(i) Rank of transpose of a matrix is same as that of the matrix.

\[ \rho(A^T) = \rho(A) \]

(ii) Rank of product of two matrices is less than or equal to minimum of the rank of individual matrices.

\[ \rho(AB) \leq \min[\rho(A), \rho(B)] \]
Given: A shape $ABC$ and its mirror image $A_1B_1C_1$ in a 2D coordinate axis.

Let, the point $A$ is $(x, y)$.

$$A = \begin{bmatrix} x \\ y \end{bmatrix}$$

which is in the first quadrant,

The mirror image $A_1$ is formed in fourth quadrant.

$$A_1 = \begin{bmatrix} x \\ -y \end{bmatrix}$$

Let the transformation matrix is $(T)$.

Then, $A_1 = AT$

From option (A):

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix} \neq \begin{bmatrix} x \\ -y \end{bmatrix}$$

Hence, option (A) is incorrect.

From option (B):

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ -x \end{bmatrix} \neq \begin{bmatrix} x \\ -y \end{bmatrix}$$

Hence, option (B) is incorrect.

From option (C):

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -x \\ y \end{bmatrix} \neq \begin{bmatrix} x \\ -y \end{bmatrix}$$

Hence, option (C) is incorrect.

From option (D):

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -y \end{bmatrix}$$

Hence, the correct option is (D).

---

**Engineering Mathematics: Linear Algebra**

**Given:** $N$ is a $3 \times 3$ matrix with real number entries and $N^2 = 0$.

For a nilpotent matrix, $N^k = 0$ where, $k =$ Positive integer

Therefore, matrix $N$ is a nilpotent matrix.

Determinant and trace of nilpotent matrix are zero.

Let Eigen value of matrix $N$ be $\lambda_1, \lambda_2$ and $\lambda_3$.

Trace of matrix = Sum of Eigen values

$$0 = \lambda_1 + \lambda_2 + \lambda_3 \quad \ldots (i)$$

Determinant of matrix

$$0 = \lambda_1 \times \lambda_2 \times \lambda_3 \quad \ldots (ii)$$

Only option (A) satisfies the criteria in equations (i) and (ii).

Hence, the correct option is (A).

---

**Key Point**

Determinant and trace of nilpotent matrix are zero.

**Given:** $3x + 2ky = -2$ and $kx + 6y = 2$

where, $k$ is a real constant.

Condition for infinite solutions is given by,

$$\rho(A) = \rho(A:B) < \text{No. of unknowns}$$

Number of unknowns = 2

So,

$$\rho(A) = \rho(A:B) < 2 \quad \ldots (i)$$

By elementary transformation,

$R_2 \rightarrow R_1 + R_2,$

$$[A:B] = \begin{bmatrix} 3 & 2k & -2 \\ k & 6 & 2 \end{bmatrix}$$

Hence, the correct option is (D).
For condition stated in equation (i) to be satisfied,

\[ k + 3 = 0 \implies k = -3 \]

and \[ 6 + 2k = 0 \implies k = -3 \]

Hence, the correct option is (C).
3.1 Figure shows a balanced ac bridge is excited by a voltage source of fixed frequency. The impedance of unknown arm $Z$ is

(A) $R$ and $L$ in series.
(B) $R$ and $C$ in series.
(C) $R$ and $C$ in parallel.
(D) $R$ and $L$ in parallel.

3.2 In bridge circuit shown in figure, $CD$ is an unknown resistance of approximately 1500 $\Omega$. The galvanometer has a resistance of 90 $\Omega$ and a sensitivity of 0.05 $\mu$A/mm. If a galvanometer gives a deflection of 60 mm, when the adjustable arm $(AD)$ is set at 150 $\Omega$. Find how much does the adjustable arm differ from its true balance value

(A) 0.5
(B) 0.056
(C) 0.2
(D) 0.01

3.3 The magnitude of impedance is ___ $\Omega$.

3.4 The power factor of the unknown impedance is ____ lead.
1995  IIT Kanpur

3.5  Find the excitation frequency (Hz) and the value of resistance $R_2$ in the a.c. bridge shown in figure under balance condition. The circuit component values are given as

$$R_1 = 100 \text{ k}\Omega, \quad R_3 = R_4 = 100 \text{ k}\Omega, \quad C_i = 2C_2 = 10 \text{ nF}.$$  

(A) 50 Hz, 100 k\Omega  
(B) 100 Hz, 200 k\Omega  
(C) 159.2 Hz, 200 k\Omega  
(D) 200 Hz, 100 k\Omega

1996  IISc Bangalore

3.6  The equations under balanced condition for a bridge are:

$$R_1 = \frac{R_2 R_3}{R_4} \quad \text{and} \quad L_1 = R_2 R_3 C_4$$

where, $R_1$ and $L_1$ are respectively unknown resistance and inductance.

In order to achieve faster balance which of the following variables are chosen?  
(A) $R_2$ and $R_3$  
(B) $R_2$ and $C_4$  
(C) $R_4$ and $C_4$  
(D) $R_3$ and $C_4$

3.7  In the bridge circuit shown in figure when $\frac{X_C}{R} = 1$, then voltmeter reads

3.8  In an ac bridge circuit shown in figure, at balance the parameters of the coil are calculated to be 50 mH and 20 \Omega.

Another coil is connected in parallel to the previous coil and balance is restored by changing $R_2$ to 2.5 k\Omega. The parameters of the new coil are

(A) 25 mH, 20 \Omega  
(B) 50 mH, 20 \Omega  
(C) 50 mH, 10 \Omega  
(D) 100 mH, 40 \Omega

3.9  Match the parameter with the type of bridge used for measurement

<table>
<thead>
<tr>
<th>List - I</th>
<th>List - II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Low value of $R$</td>
<td>P</td>
</tr>
<tr>
<td>(ii) High value of $Q$</td>
<td>Q</td>
</tr>
<tr>
<td>(iii) Inductance $L$</td>
<td>R</td>
</tr>
<tr>
<td>(iv) Capacitance $C$</td>
<td>S</td>
</tr>
</tbody>
</table>
1.33

**GATE ACADEMY**

**Electrical & Electronics Measurement: Measurement of Resistance & AC Bridges**

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) iv</td>
<td>iii</td>
<td>i</td>
<td>ii</td>
</tr>
<tr>
<td>(B) iii</td>
<td>i</td>
<td>ii</td>
<td>iv</td>
</tr>
<tr>
<td>(C) iv</td>
<td>ii</td>
<td>i</td>
<td>iii</td>
</tr>
<tr>
<td>(D) i</td>
<td>iv</td>
<td>iii</td>
<td>ii</td>
</tr>
</tbody>
</table>

### 1998 IIT Delhi

**3.10** For the circuit given in figure. The unbalanced voltage $e_o$ is

![Circuit Diagram]

(A) $e_o = IRX$

(B) $e_o = \frac{IRX}{2}$

(C) $e_o = 2IRX$

(D) $e_o = IR(1+X)$

**3.11** Match the measured variables with the bridge

<table>
<thead>
<tr>
<th>List - I</th>
<th>List - II</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Self inductance</td>
<td>(p) Anderson</td>
</tr>
<tr>
<td>(b) Capacitance</td>
<td>(q) Wien-Robinson</td>
</tr>
<tr>
<td>(c) Mutual inductance</td>
<td>(r) Campbell</td>
</tr>
<tr>
<td>(d) Frequency</td>
<td>(s) Schering</td>
</tr>
</tbody>
</table>

(A) a - s, b - p, c - q, d - r

(B) a - p, b - s, c - r, d - q

(C) a - r, b - s, c - p, d - q

(D) a - p, b - q, c - r, d - s

**1999 IIT Bombay**

**3.12** The ac bridge in figure remains balance if $Z$ comprises of

![AC Bridge Diagram]

(A) resistance only.

(B) capacitance only.

(C) resistance and capacitance in parallel.

(D) resistance and inductor in series.

**3.13** The bridge circuit in figure is balanced, the value of current $I$ is

![Bridge Circuit Diagram]

(A) 2 mA

(B) 4 mA

(C) 5 mA

(D) 6 mA

### 2000 IIT Kharagpur

**Common Data for Questions 3.14 & 3.15**

The bridge circuit shown in figure is balanced at a supply frequency $\omega = 1000 \text{ rad/sec}$ for the values $R_1 = 1 \text{ k}\Omega$, $R_2 = 2 \text{ k}\Omega$, $C_2 = 0.1 \mu\text{F}$ and $C_3 = 0.05 \mu\text{F}$.
The unknown element $x$ may be a pure inductance, a pure capacitance or a pure resistance.

3.14 The element $x$ identify by a
(A) pure capacitor.
(B) pure inductor.
(C) series combination of resistor and inductor.
(D) series combination of resistor and capacitor.

3.15 The value of $x$ and $R_x$ is
(A) $10^{-6} \text{F}, 0.2 \times 10^3 \Omega$
(B) $10^{-6} \text{F}, 2 \times 10^3 \Omega$
(C) $2 \text{mH}, 1 \times 10^3 \Omega$
(D) $10^{-4} \text{F}, 0.2 \times 10^4 \Omega$

3.16 In the context of low resistances, identify the incorrect statement
(A) Standard low resistance have four terminal construction.
(B) Kelvin bridge is used for measuring low resistance, precision depends on detector sensitivity.
(C) A pair of ratio arms in Kelvin double bridge for measuring the low resistance eliminates the error due to thermo emf.
(D) Low resistances used for ammeter shunts are usually made with a suitable number of plates of large area and all the plates are connected in parallel.

3.17 Kelvin double bridge is best suited for the measurement of
(A) resistance of very low value.
(B) low value of capacitance.
(C) resistance of very high value.
(D) high value capacitance.

3.18 Wien bridge is best suited for the measurement of
(A) frequency
(B) capacitor
(C) inductor
(D) resistor

3.19 For the bridge circuit in figure, the voltages are
\[ V_1 = \sqrt{2} \cos 1000t \ \text{V} \]
\[ V_2 = 2 \cos(1000t + 45^\circ) \text{ and } V_d = 0 \]
The value of $R = 100 \ \Omega$. Then $Z_x$ will be

3.20 The output resistance across the terminals 1 and 2 of the DC bridge in figure is
3.21 The resistance values of the bridge circuit shown in figure are $R_1 = R_2 = R_3 = R$ and $R_4 = R + \Delta R$. The bridge is balanced by introducing a small voltage $v$. The value of $\Delta R$ is

\[
\frac{v}{E} \quad \text{(A)} \quad \frac{Rv}{E - v} \quad \text{(B)} \quad \frac{2vR}{E - 2v} \quad \text{(C)} \quad \frac{Rv}{E + 2v} \quad \text{(D)}
\]

3.22 In the AC bridge shown in figure, the detector $D$ reads zero. Then $Z$ is made of

(A) $50$ mH in parallel with $50$ $\Omega$.
(B) $50$ mH in series with $50$ $\Omega$.
(C) $50$ nF in series with $10$ k$\Omega$.
(D) $50$ nF in parallel with $5$ k$\Omega$.

3.23 The bridge shown in figure is balanced. The dissipation factor for the unknown lossy capacitor $R_x + \frac{1}{j\omega C_x}$ is

(A) $\omega R_x C_3$
(B) $\frac{\omega R_x^2 C_3}{R_2}$
(C) $\omega R_x C_3$
(D) $\frac{R_1}{R_2}$

3.24 If the supply voltage to the bridge is decreased by 5%, the sensitivity of the measurement system
(A) decreases by 5%.
(B) does not change.
(C) increases by 5%.
(D) increases by 20%.

3.25 Consider the AC bridge shown below. If $\omega R C = 1$ and $\Delta C / C < 0.01$, then ratio $\left| \frac{V_o}{V_i} \right|$ is approximately equal to

(A) $50$ mH in parallel with $50$ $\Omega$.
(B) $50$ mH in series with $50$ $\Omega$.
(C) $50$ nF in series with $10$ k$\Omega$.
(D) $50$ nF in parallel with $5$ k$\Omega$. 
Consider the AC bridge shown in the figure below, with $R$, $L$ and $C$ having positive finite values.

Then

(A) $V_o = 0$ if $\omega L = -\frac{1}{\omega C}$

(B) $V_o = 0$ if $L = C$

(C) $V_o = 0$ if $R = \frac{1}{\omega \sqrt{LC}}$

(D) $V_o$ cannot be made zero

In the Wheatstone bridge shown below the galvanometer $G$ of current sensitivity of $1 \mu A/mm$, a resistance of $2.5 \text{ k}\Omega$ and a scale resolution of $1 \text{ mm}$. Let $\Delta R$ be minimum increase in $R$ from its nominal value of $2 \text{ k}\Omega$ that can be detected by this bridge.

When $R$ is $2 \text{ k}\Omega + \Delta R$, $V_{AN}$ is

(A) $6 \text{ V}$

(B) $6.0024 \text{ V}$

(C) $6.0038 \text{ V}$

(D) $6.005 \text{ V}$

The value of $\Delta R$ is approximately

(A) $2.8 \text{ \Omega}$

(B) $3.4 \text{ \Omega}$

(C) $5.2 \text{ \Omega}$

(D) $12 \text{ \Omega}$

If an ac bridge circuit shown below is balanced, the element $Z$ can be

(A) pure capacitor

(B) pure inductor

(C) $R-L$ series combination

(D) $R-L$ parallel combination

In Wheatstone bridge shown below when the resistance $R_i$ increases $1 \text{ k}\Omega$, the current through galvanometer is

(A) $1.25 \mu A$

(B) $2.5 \mu A$

(C) $12.5 \mu A$

(D) $25 \mu A$
3.31 The bridge method commonly used for finding mutual inductance is
(A) Heaviside Campbell bridge
(B) Schering bridge
(C) De Sauty bridge
(D) Wien bridge

3.32 The resistance and inductance of an inductive coil are measured using an AC bridge as shown in figure. The bridge is to be balanced by varying the impedance $Z_2$.

For obtaining balance $Z_2$ should consist of element
(A) $R$ and $C$  
(B) $R$ and $L$  
(C) $L$ and $C$  
(D) Only $C$

3.33 If the deflection of the galvanometer in the bridge circuit shown in the figure is zero, then the value of $R_x$ in ohms is _______ $\Omega$.

3.34 The bridge most suited for measurement of a four-terminal resistance in the range of $0.001 \Omega$ to $0.1 \Omega$ is
(A) Wien’s bridge.
(B) Kelvin double bridge.
(C) Maxwell’s bridge.
(D) Schering bridge.

3.35 In an a.c. bridge, shown in the figure, $R = 10^4 \Omega$ and $C = 10^{-7} F$. If the bridge is balanced at a frequency $\omega_0$, the value of $\omega_0$ in rad/s is _______.

3.36 The unbalanced voltage of the Wheatstone bridge, shown in the figure is measured using a digital voltmeter having infinite input impedance and a resolution of 0.1 mV. If $R = 1000 \Omega$, then the minimum value of $\Delta R$ in $\Omega$ to create a detectable unbalanced voltage is______.
3.37 The inductance of a coil is measured using the bridge shown in the figure. Balance \((D = 0)\) is obtained with 
\[C_1 = 1 \text{ nF}, \quad R_1 = 2.2 \text{ M}\Omega, \quad R_2 = 22.2 \text{ k}\Omega, \quad R_4 = 10 \text{ k}\Omega.\]

The value of the inductance \(L_x\) (in mH) is _____.

\[
\frac{\text{V}_1}{R_1} = \frac{\text{V}_2}{R_2} = \frac{\text{V}_3}{R_4} = D
\]

\[
\text{D} = \frac{C_1}{R_1} = \frac{C_2}{R_2} = \frac{1}{R_4} = \frac{1}{R_3}
\]
### Answers

<table>
<thead>
<tr>
<th>Q</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>C</td>
<td>3.2</td>
<td>B</td>
<td>3.3</td>
<td>7961.67</td>
<td>3.4</td>
<td>0.0314</td>
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<tr>
<td>3.6</td>
<td>C</td>
<td>3.7</td>
<td>A</td>
<td>3.8</td>
<td>C</td>
<td>3.9</td>
<td>A</td>
</tr>
<tr>
<td>3.11</td>
<td>B</td>
<td>3.12</td>
<td>C</td>
<td>3.13</td>
<td>D</td>
<td>3.14</td>
<td>A</td>
</tr>
<tr>
<td>3.16</td>
<td>C</td>
<td>3.17</td>
<td>A</td>
<td>3.18</td>
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<td>3.21</td>
<td>C</td>
<td>3.22</td>
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### Explanations

#### Concept of Wheatstone Bridge

#### Concept of Schering Bridge

#### Concept of AC Bridge (Part - I)

#### Concept of Unmodified De Sauty's Bridge

#### Concept of AC Bridge (Part - II)

#### Concept of Modified De Sauty's Bridge

#### Concept of AC Bridge (Part - III)

#### Concept of AC Bridge (Part - IV)

#### Concept of AC Bridge (Part - V)

### Method 1

The above figure can be represented as,
Since, the bridge is balance,

Condition for bridge balance:

(i) Product of opposite arm impedance should be equal i.e.

\[ Z_{AB}Z_{BC} = Z_{AB}Z_{CD} \]

(ii) The detector current should be zero.

i.e. \[ I = 0 \]

\[ R_1R_3 = Z(R_2 + j\omega L_2) \]

\[ Z = \frac{R_1R_3}{R_2 + j\omega L_2} \]

\[ Z = \frac{R_1R_3}{R_2 + j\omega L_2} - j\omega \frac{R_1R_3}{R_2 + j\omega L_2} \]

\[ Z = R_{eq} - j\omega X_{eq} \quad \text{(by rationalizing)} \]

\[ Z = \frac{R_1R_3}{R_2 + j\omega L_2} = \frac{R_{eq}}{1 + j\omega \frac{L_2}{R_2}} \]

So, we got equation (i) with help of rationalization and equation (ii) without rationalizing, now going by options,

(A) \( R \) and \( L \) in series

\[ Z_{eq} = R + j\omega L \]

Neither of equation (i) and (ii) matches.

(B) \( R \) and \( C \) in series

\[ Z_{eq} = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C} \]

In equation (i), \( j \) and \( \omega \) appears in multiplication and in series combination of \( R \) and \( C \) they appear in division so not possible and the equation does not matches with equation (ii) also.

(C) \( R \) and \( C \) in parallel

\[ Z_{eq} = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + \frac{j\omega C}{R}} \]

\[ Z_{eq} = R \parallel \frac{1}{j\omega C} = \frac{R}{1 + j\omega \tau} \]

\[ \tau = \text{Time constant (sec)} \]

From equation (ii),

\[ Z_{eq} = \frac{R}{1 + j\omega \frac{L_2}{R_2}} \]

Unit of \( \frac{R}{L_2} = \text{seconds} \)

So, option (C) is satisfied.

(D) \( R \) and \( L \) in parallel

\[ Z_{eq} = \frac{R \times j\omega L}{R + j\omega L} \]

\[ Z_{eq} = \frac{\omega^2 L^2 R}{R^2 + (\omega L)^2} + \frac{j\omega R^2 L}{R^2 + (\omega L)^2} \]

\[ Z_{eq} = R_{eq} + j\omega X_{eq} \quad \text{[By rationalizing]} \]

\[ Z_{eq} = \frac{R}{1 + \frac{1}{j\omega L}} = \frac{R}{1 - j\frac{R}{\omega L}} \]

Neither of equation (i) and (ii) is satisfied.

Hence, the correct option is (C).
Method 2

Given bridge circuit is shown below,

![Fig. (a)](image)

The above bridge is the standard Maxwell inductance capacitance bridge as,

![Fig. (b)](image)

Comparing the above two figures, fig. (a) and (b).

We have, the unknown impedance $Z$ is the parallel combination of $R$ and $C$.

Hence, the correct option is (C).

Method 3

Given bridge circuit is shown below,

In case of balanced bridge $V_A = V_B$,

$$V_A = \frac{V \times Z}{Z + R_1} \quad \text{and} \quad V_B = \frac{V \times R_3}{R_3 + R_2 + j\omega L_2}$$

During balance condition, $I_D = 0$ hence $R_1$ and $Z$ are in series and $R_2$, $L_2$ and $R_3$ are in series so in order to calculate $V_A$ and $V_B$ we applied voltage division rule, so in balance condition

$$V_A = V_B$$

$$\frac{V \times Z}{Z + R_1} = \frac{V \times R_3}{R_3 + R_2 + j\omega L_2}$$

$$\frac{Z}{Z + R_1} = \frac{R_3}{R_3 + R_2 + j\omega L_2}$$

Applying componendo and dividendo,

$$\frac{Z}{Z - Z} = \frac{R_3}{R_2 + R_3 + j\omega L_2 - R_1}$$

$$\frac{Z}{R_4} = \frac{R_3}{R_2 + j\omega L_2}$$

$$Z = \frac{R_2 R_3}{R_2 + j\omega L_2} = \frac{R_2}{1 + j\omega L_2}$$

$$\frac{R_2}{R_2} = \frac{\Omega \times \Omega}{\Omega} = \Omega \quad [\text{Unit} : \Omega]$$

$$\frac{L_2}{R_2} = \frac{H}{\Omega} = \text{seconds} \quad [\text{Unit} : \text{sec}]$$

$$Z = \frac{R_2 R_3}{R_2 + j\omega L_2}$$

Thus, it is equivalent to parallel combination of $R$ and $C$.

Hence, the correct option is (C).
3.2 (B)

Given bridge circuit is shown below,

\[\Delta R = 150.056 - 150 = 0.056 \, \Omega\]

Hence, the correct option is (B).

3.3 7961.67

Given balance bridge circuit is shown below,

Given:
- \(C_1 = 0.001 \, \mu F\)
- \(C_3 = 0.01 \, \mu F\)
- \(R_1 = 5000 \, \Omega\)
- \(R_2 = 2500 \, \Omega\)

Since, the above bridge is balance.

Condition for bridge balance:
Product of opposite arm impedance should be equal i.e.,
\[
Z_{AD}Z_{BC} = Z_{AB}Z_{CD}
\]

\[
R_1 \times \frac{1}{j\omega C_1} \times Z_X = R_2 \times \frac{1}{j\omega C_3}
\]

\[
R_1 Z_X = \frac{R_2(1 + j\omega R_1 C_1)}{j\omega C_1 R_1}
\]

\[
Z_X = \frac{R_2 C_1}{C_3} + \frac{1}{j\omega \left( C_3 \times \frac{R_1}{R_2} \right)}
\]

From the given data,
\[
Z_X = \frac{2500 \times 0.001 \times 10^{-6}}{0.01 \times 10^{-6}} + \frac{1}{j(2\pi \times 1000) \left( \frac{0.01 \times 10^{-6} \times 5000}{2500} \right)}
\]
\[ Z_X = 7961.67 \angle -88.2^\circ \]

Hence, the magnitude of impedance is \( 7961.67 \Omega \).

From the previous question,
\[ Z_X = 7961.67 \angle -88.2^\circ \]

Power factor (\( \cos \phi \)) is given by,
\[ \cos \phi = \cos(88.2) = 0.0314 \text{ lead} \]

Hence, the power factor of the unknown impedance is 0.0314 lead.

Given balance bridge circuit is shown below,

Equating the imaginary term on both side,

\[ \omega R_1 C_1 R_4 + \omega R_2 C_2 R_4 = \omega C_1 R_2 R_3 \]
\[ R_1 C_1 R_4 + R_2 C_2 R_4 = C_1 R_2 R_3 \]
\[ R_2 = \frac{R_1 C_1 R_4}{R_1 C_1 - R_4 C_2} \]

From the given data,
\[ R_2 = \frac{100 \times 10^3 \times 10 \times 10^{-9} \times 100 \times 10^3}{(100 \times 10^3 \times 10 \times 10^{-9} - 100 \times 10^3 \times 5 \times 10^{-9})} \]
\[ R_5 = 200 \, k\Omega \]

Equating the real term on both side of equation (i),
\[ R_4 - \omega^2 R_1 C_1 R_2 R_3 C_2 = 0 \]
\[ \omega^2 = \frac{1}{R_1 C_1 R_2 C_2} \]
\[ \omega = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}} \]

and
\[ f = \frac{1}{2\pi \sqrt{R_1 C_1 R_2 C_2}} \]
\[ f = \frac{1}{2\pi \sqrt{100 \times 10^3 \times 10 \times 10^{-9} \times 200 \times 10^3 \times 5 \times 10^{-9}}} \]

\[ f = 159.2 \, Hz \]

Hence, the correct option is (C).

Key Points

**Wien's Bridge**:

(i) Wien bridge is used for measurement of wide range of frequency in the MHz range.

(ii) It is suitable for sinusoidal waves only because the bridge can be balanced only at a particular frequency.
The frequency of oscillation is given by,

\[ f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}} \]

If \( R_1 = R_2 = R, \ C_1 = C_2 = C \) then \( f = \frac{1}{2\pi RC} \).

**Method 1**

The above equation is used to find the unknown inductance \( L \) having resistance \( R \) in Maxwell \( L-C \) bridge.

In the Maxwell inductance bridge, the variable elements are \( R_4 \) and \( C_4 \).

From equations (i) and (ii),

It is clear that firstly we vary \( C_4 \) so that we get a required (fixed) inductance (\( L_1 \)).

After fixing the value of inductance, by varying \( R_4 \) we vary the value of \( R_1 \) and select a certain value.

Hence, the correct option is (C).

**Key Point**

Maxwell inductance capacitance bridge:

At balance condition

\[ Z_1 Z_4 = Z_2 Z_3 \]

\[ (R_1 + j\omega L_1) \left( \frac{R_4}{1 + j\omega R_4 C_4} \right) = R_2 R_3 \]

\[ R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega R_4 R_4 C_4 \]

Comparing real and imaginary part,

For real part, \( R_1 R_4 = R_2 R_3 \)

\[ R_1 = \frac{R_2 R_3}{R_4} \]

For imaginary part,

\[ L_1 = R_2 R_4 C_4 \]

**Method 2**

\[ R_1 = \frac{R_2 R_3}{R_4} \]  \( \ldots \) (i)

\[ L_1 = R_2 R_4 C_4 \]  \( \ldots \) (ii)

From equations (i) and (ii),

Uncommon terms are \( R_4 \) and \( C_4 \).

So, to achieve faster balance the required variables are \( R_4 \) and \( C_4 \).

Hence, the correct option is (C).
Given bridge circuit is shown below,

![Bridge Circuit](image)

Given:
\[
\frac{X_C}{R} = 1
\]

\[
X_C = R \quad \ldots (i)
\]

**Calculation of voltmeter reading:**

From figure,
\[
V_D = 10 \times \frac{R}{R + R} = 5 \text{ V} \quad \text{[By VDR]}
\]
\[
V_B = 10 \times \frac{-jX_C}{R - jX_C} \quad \text{[By VDR]}
\]

From equation (i),
\[
V_B = 10 \times \frac{-jR}{R - jX_C} = 7.071 \angle -45^0
\]

The voltmeter reading is given by,
\[
V = V_D - V_B
\]
\[
V = 5 - 7.071 \angle -45^0 = 5 \angle 90^0
\]

Hence, the correct option is (A).

---

Given bridge circuit is shown below,

![Bridge Circuit](image)

If another coil is connected in parallel to the previous coil then balanced is achieved by changing \( R_2 = 2.5 \text{ k}\Omega \)

The modified figure is shown below,

![Modified Bridge Circuit](image)

Apply bridge balance condition i.e. product of opposite arm impedances are equal as,
\[
Z_{eq} = 20 \Omega + j\omega 50 \text{ mH}
\]
\[
\left[ Z_{eq} \right] \times Z_4 = Z_3 \times Z_2
\]
\[
\left[ Z_{eq} \right]_{new} \times Z_4 = Z_3 \times Z_2 / 2
\]

So, \( Z_2 \) is reduced by factor of 2 therefore to restore balance
\[
\left[ Z_{eq} \right]_{new} = \frac{Z_{eq}}{2}
\]
\[
\left[ Z_{eq} \right]_{new} = \frac{20 \Omega + j\omega 50 \text{ mH}}{2}
\]
\[ Z_{eq_{new}} = 10 \Omega + j0.25 \text{ mH} \]

Now, \( 10 \Omega + j0.25 \text{ mH} = (20 + j0.50) || Z_{coil} \)

So, \( Z_{coil} = 10 \Omega + j0.50 \text{ mH} \)

Hence, the correct option is (C).

3.9 \( \boxed{\text{(A)}} \)

**Kelvin double bridge** : Kelvin double bridge is used for measurement of low value of resistance.

Hence, (i)-R

**Maxwell’s-Wien bridge / Maxwell L-C bridge** : This bridge is used for measurement of inductance (specially for inductance of medium quality factor).

Hence, (iii)-Q

**Hay bridge** : Hay bridge is used for measurement of inductance of high quality factor \((Q > 10)\).

Hence, (ii)-S

**Schering bridge** : Schering bridge is used for measurement of capacitance and dissipation factor.

Hence, (iv)-P

Hence, the correct option is (A).

\[ \text{Key Point} \]

**Different methods for measurement of } R, L \text{ and } C :**

(i) **Measurement of resistance** :

(a) Low resistance measurement method

1. Kelvin’s double bridge
2. Potentiometer
3. A-V/V-A method

(b) Medium resistance measurement method :

1. Wheatstone bridge
2. A-V/V-A method
3. Carey Foster bridge
4. Ohm meter
5. Substitution method

(c) High resistance measurement method

1. Megger
2. Loss of charge method
3. V/I method

(ii) **Measurement of inductance** :

(a) Maxwell L-C bridge \((1 < QF < 10)\)

(b) Hay bridge \((QF > 10)\)

(c) Anderson’s bridge \((QF < 1)\)

(d) Owen’s bridge

(iii) **Measurement of capacitance** :

(a) Schering bridge
(b) De-Sauty bridge

3.10 \( \boxed{\text{(B)}} \)

Given circuit is shown below,

From figure,

\[ I_1 = I_2 = I \times \frac{R + R(1 + X)}{R + R(1 + X) + R + R(1 + X)} \]

[By CDR]

\[ I_1 = I_2 = \frac{I}{2} \] …(i)

Applying KVL in the above shown loop,

\[ V_{DB} = e_0 = I_2 R(1 \times X) - I_1 R \]

From equation (i),

\[ e_0 = \frac{I}{2} R(1 + X) - \frac{I}{2} R = \frac{IRX}{2} \]

Hence, the correct option is (B).
3.11  (B)

Schering bridge: This bridge is used for measurement of self-capacitance.
Hence, (b) - (s)

Wien-Robinson: This bridge is used for measurement of frequency.
Hence, (d) - (q)

Campbell: Campbell bridge is used for measurement of mutual inductance.
Hence, (c) - (r)

Anderson bridge: Anderson bridge is used for measurement of self-inductance (Low quality factor).
Hence, (a) - (p)
Hence, the correct option is (B).

3.12 (C)

Given balance bridge is shown below,

**Method 1**

Since, the bridge is balance,

**Condition for bridge balance:**

\[
Z_{\omega}Z_{\omega} = Z_{ac}Z_{bd}
\]

\[
R_1 \times \frac{1}{j\omega C_1} = Z \times \left( R_2 + \frac{1}{j\omega C_2} \right)
\]

\[
\frac{R_1}{j\omega C_1} = \frac{Z(1 + j\omega R_1 C_2)}{j\omega C_2}
\]

\[
\frac{C_2}{C_1} R_1 = Z(1 + j\omega R_1 C_2)
\]

Thus, impedance \( Z \) is the parallel combination of resistance and capacitance.
Hence, the correct option is (C).

**Method 2**

The above bridge can be modified as,

This is the standard Schering bridge.
where, \( Z \) is the parallel combination of resistance and capacitance.
Hence, the correct option is (C).

3.13 (D)

Given balance bridge circuit is shown below,
Since, the above bridge is balance.

**Condition for bridge balance:**

Product of opposite arm impedance should be equal i.e.

\[ Z_{AD} Z_{BC} = Z_{AB} Z_{CD} \]

1 kΩ × 2 kΩ = \( Z_{AB} \times 4 \) kΩ

\[ Z_{AB} = 0.5 \) kΩ \]

[Internal resistance of 2 V]

Current through branch \( AB \) is given by,

\[ I_{AB} = \frac{V_{AB}}{Z_{AB}} = \frac{2}{0.5 \times 10^3} = 4 \) mA \ ...(i)

From figure,

\[ I_{AB} = I \times \frac{Z_{ADC}}{Z_{ADC} + Z_{ABC}} \]

4 mA = \( I \times \frac{1 \) kΩ + 4 kΩ}{1 kΩ + 4 kΩ + 0.5 kΩ + 2 kΩ

\( I = 6 \) mA

Hence, the correct option is (D).

**Method 1**

The above bridge is the standard Schering bridge.

Given : \( R_1 = 1 \) kΩ, \( R_2 = 2 \) kΩ,

\[ C_2 = 0.1 \) μF, \( C_3 = 0.05 \) μF

**Method 2**

Comparing the above both figures, it is clear that the element \( x \) must be a pure capacitor.

Hence, the correct option is (A).

**Condition for bridge balance:**

(i) Product of opposite arm impedance should be equal i.e.

\[ Z_{AD} Z_{BC} = Z_{AB} Z_{CD} \] \ ...(i)

(ii) The detector current should be zero.

i.e. \( I = 0 \)

From equation (i),

\[ R_1 \times \frac{1}{jωC_2} = \frac{R_2}{1 + jωR_2C_2} \]

\[ R_4 + x = \frac{R_1}{jωC_3} \times \frac{(1 + jωR_2C_2)}{R_2} \]
The value of resistance $R_4$ is given by,
\[ R_4 = \frac{C_2 \times R_1}{C_3} \]
\[ R_4 = \frac{0.1 \times 10^{-6}}{0.05 \times 10^{-6}} \times 1 \times 10^3 = 2000 \Omega \]

From the given data,
\[ x = \frac{1}{j \omega \times 2 \times 10^3 \times 0.05 \times 10^{-6}} \]
\[ + \frac{1 \times 10^3 \times 0.1 \times 10^{-6}}{0.05 \times 10^{-6}} \]
\[ x = \frac{1}{j \omega \times 0.1 \times 10^{-3}} \]

Thus, the element $x$ behaves as a pure capacitor.
Hence, the correct option is (A).

From the previous question,

The above bridge is the standard Schering bridge as,

The value of $x$ is given by,
\[ x = \frac{1}{j \omega \times 0.1 \times 10^{-3}} \]

[From previous solution]

Hence, the correct option is (D).

3.16  (C)

(i) The standard low resistance ($R < 1 \Omega$) is represented by four terminals as,

So, the option (A) is correct.

(ii) Kelvin’s double bridge is used for measurement of low resistance ($R < 1 \Omega$).

So, the option (B) is correct.

(iii) In the Kelvin’s double bridge reversible switch is used to reduce the effect of thermoelectric emfs produced at the junction.

So, the option (C) is incorrect.

(iv) Low resistance are used for ammeter shunts. These are usually made with a suitable number of plates of large area and all the plates are connected in parallel.

So, the option (D) is correct.

Hence, the correct option is (C).

3.17  (A)

Kelvin double bridge is used for measurement of low resistance ($R < 1 \Omega$). It is used to measure winding resistance of transformer, generator, motor and earth conductor resistance.
1.50

To pic Wise GATE Solutions [IN]

Fig. Circuit diagram

Hence, the correct option is (A).

3.18 (A)

Wien bridge is best suited for measurement of frequency.

Frequency of oscillation is given by,

\[ f = \frac{1}{2\pi\sqrt{RC}} \]

If \( R_1 = R_3 = R \) and \( C_1 = C_3 = C \)

\[ f = \frac{1}{2\pi\sqrt{RC}} \]

Wien bridge is used for measurement of fundamental of frequency. If some harmonics are present, then this bridge is not used.

Hence, the correct option is (A).

Given bridge circuit is shown below,

Given : \( V_1 = \sqrt{2}\cos(1000t) \)

\( V_2 = 2\cos(1000t + 45^\circ) \)

\( V_d = 0, \quad \omega = 1000 \)

\( V_{1ms} = \frac{\sqrt{2}}{\sqrt{2}} = 1\angle 0^\circ \text{ V} \)

\( V_{2ms} = \frac{2\angle 45^\circ}{\sqrt{2}} = \sqrt{2}\angle 45^\circ \)

The above figure can be represented as,

Since, the bridge is balance.

**Condition for bridge balance** :

(i) The product of opposite arm impedance are equal i.e.,

\[ Z_{AD}Z_{BC} = Z_{AB}Z_{CD} \quad \ldots (i) \]

(ii) The potential at point \( B \) and \( D \) are same i.e. \( V_B = V_D \)

From figure,

\( V_{AD} = V_{AB} = 1\angle 0^\circ \text{ V} \)

\( I_2 = \frac{V_{AB}}{100} = \frac{1\angle 0^\circ}{100} = 0.01 \text{ Amp} \)
Similarly, \( V_{BC} = V_{DC} = \sqrt{2} \angle 45^\circ \ V \)

and \( Z_x = \frac{V_{BC}}{I_2} = \frac{\sqrt{2} \angle 45^\circ}{0.01} = 100 \sqrt{2} \angle 45^\circ \)

\( Z_x = 100 + j100 = R + j\omega L \)

Here, \( 1000L = 100 \)

\( L = 0.1 \text{ H} = 100 \text{ mH} \)

where, \( R = 100 \ \Omega, \ \omega L = 100. \)

Thus the impedance \( Z_x \) will be 100 \( \Omega \) resistor in series with 100 mH inductor.

Hence, the correct option is (A).

**3.20 (B)**

Given bridge circuit is shown below,

For output resistance across terminals 1 and 2, the independent source should be replaced by their internal resistance,

\[
R_{12} = \frac{30 \times 20}{30 + 20} + \frac{25 \times 25}{25 + 25} = 24.5 \text{ k}\Omega
\]

Hence the correct option is (B).

---

**Given bridge circuit is shown below,**

For output resistance across terminals 1 and 2, the independent source should be replaced by their internal resistance,

\[
R_{12} = \frac{30 \times 20}{30 + 20} + \frac{25 \times 25}{25 + 25} = 24.5 \text{ k}\Omega
\]

Hence the correct option is (B).

---

**Given :** Bridge is balance so the current through the detector is zero, i.e. open circuit.

Applying the bridge balance condition, product of opposite arm impedance should be equal i.e.

\[
Z_{AB} Z_{CD} = Z_{AD} Z_{BC}
\]

\[
R \times Z_{CD} = R \left( R + \Delta R \right)
\]

\[
Z_{CD} = R + \Delta R
\]

[Voltage (\( v \)) has internal resistance of \( \Delta R \) ]

From figure,

\[
I_{1} = \frac{E}{Z_{BC} + Z_{CD}}
\]

\[
I_{1} = \frac{E}{R + \Delta R + R + \Delta R}
\]
\[ I_1 = \frac{E}{2(R + \Delta R)} \]  
\[ V_{CD} = E \times \frac{Z_{CD}}{Z_{BC} + Z_{CD}} \]
\[ V_{CD} = E \times \frac{R \times \Delta R}{(R+\Delta R) + (R+\Delta R)} \]
\[ V_{CD} = \frac{E}{2} \]

Also,

\[ 1.52 \quad \text{Topic Wise GATE Solutions [IN]} \]

\[ \text{Given : Detector reads zero i.e. bridge is balance.} \]

\[ \text{Method 1} \]

From figure,

\[ V_{CD} = v + I_1 \times R \]

From equations (i) and (ii),

\[ \frac{E}{2} = v + \frac{E}{2(R + \Delta R)} R \]

\[ \frac{E}{2} \left(1 - \frac{R}{R+\Delta R}\right) = v \]

\[ \frac{E}{2} = \frac{\Delta R}{R+\Delta R} \]

\[ 2v = 1 + \frac{R}{\Delta R} \]

\[ \frac{E}{2} \]

\[ \frac{E}{2} - 1 = \frac{R}{\Delta R} \]

\[ \Delta R = \frac{2vR}{E - 2v} \]

Hence, the correct option is (C).

3.22 \quad \text{(B)}

\[ \text{Given bridge circuit is shown below,} \]

\[ V_r = 10 \cos 314t \]

Since, the bridge is balance.

\[ \text{Condition for bridge balance :} \]

Product of opposite arm impedances are equal i.e.,

\[ Z_{AD}Z_{BC} = Z_{AB}Z_{CD} \]

\[ 10 \text{k}\Omega \times \frac{1}{j(314 \times 100 \times 10^{-6})} \times Z = 500 \times 1 \text{k}\Omega \]

\[ 10 \text{k}\Omega + \frac{1}{j(314 \times 100 \times 10^{-6})} \]

\[ 10 \text{k}\Omega \times Z \]

\[ 1 + j0.314 \]

\[ Z = 50 + j(314 \times 50 \text{ mH}) \]

Thus, the impedance Z is series combination of resistance (50 \text{ \Omega}) and inductance (50 \text{ mH}).

Hence, the correct option is (B).

\[ \text{Method 2 : Objective approach} \]
The above figure is the standard Maxwell L-C bridge as,

![Image](image.png)

The value of $R_1$ and $L_1$ is given by,

$$R_1 = \frac{R_2 R_3}{R_4} = \frac{500 \times 1 \times 10^3}{10 \times 10^3} = 50 \, \Omega$$

$$L_1 = R_2 R_3 C_4 = 500 \times 1 \times 10^3 \times 100 \times 10^{-9}$$

$$L_1 = 50 \, \text{mH}$$

Thus, $Z$ is made of $50 \, \text{mH}$ in series with $50 \, \Omega$.

Hence, the correct option is (B).

Given balanced bridge circuit is shown below,

![Image](image.png)

Since, the bridge is balance,

**Condition for bridge balance**:

Product of opposite arm impedances are equal i.e.,

$$Z_{AD} Z_{BC} = Z_{AB} Z_{CD}$$

$$R_2 \left( R_X + \frac{1}{j \omega C_X} \right) = R_1 \left( R_3 + \frac{1}{j \omega C_3} \right)$$

$$R_X = \frac{R_2 R_1}{R_3} \quad \text{and} \quad C_X = \frac{R_2 C_3}{R_1}$$

The dissipation factor for series combination of $R_X$ and $C_X$ is given by,

$$\text{Dissipation factor} = \frac{R X}{1/ \omega C} = \omega R C$$

**Key Point**

For series RC circuit,

$$\text{Quality factor} = \frac{V_C}{V_R}$$

$$\text{Dissipation factor} = \frac{1}{Q.F.} = \frac{V_R}{V_C} = \frac{IR}{IX_C}$$

Hence, the correct option is (C).

Sensitivity of the bridge is given by,

$$S_B = \frac{\theta}{\Delta R}$$

From the above expression, sensitivity of bridge is independent of supply voltage. Thus, if the supply voltage to the bridge is decreased by 5%, the sensitivity of the measurement system does not change.

Hence, the correct option is (B).
Given : \( \omega RC = 1 \) and \( \frac{\Delta C}{C} < 0.01 \)

From figure,

\[
V_{bc} = \frac{1}{j\omega(C + \Delta C)} + R
\]

... (i)  

[By VDR]

and

\[
V_{dc} = V_s \times \frac{j\omega C}{R + \frac{1}{j\omega C}}
\]

... (ii)  

[By VDR]

The output voltage is given by,

\[
V_0 = V_{bc} - V_{dc}
\]

From equations (i) and (ii),

\[
\frac{V_0}{V_s} = \frac{1}{1 + j\omega R(C + \Delta C)} - \frac{1}{1 + j\omega RC}
\]

\[
= \frac{1 + j - j - j\omega R\Delta C}{(1 + j)(1 + j + j\omega R\Delta C)}
\]

\[
= \frac{-j\omega R\Delta C}{(1 + j)(1 + j + 0)}
\]

\[
= \frac{-j\omega R\Delta C}{2}
\]

\[
= \frac{\omega RC\Delta C}{2C}
\]

Hence, the correct option is (D).

3.26 (D)
\[ V_B = V \times \frac{R}{R + R + j\omega L} \quad \text{[By VDR]} \]

\[ V_D = V \times \frac{1}{R + R + \frac{1}{j\omega C}} \quad \text{[By VDR]} \]

From figure, \[ V_0 = V_B - V_D \]

\[ V_0 = V \left[ \frac{R}{2R + j\omega L} \right] - \frac{VR}{2R + \frac{1}{j\omega C}} \]

For \( V_0 = 0 \),

\[ V \left[ \frac{R}{2R + j\omega L} \right] - \frac{VR}{2R + \frac{1}{j\omega C}} = 0 \]

\[ \frac{1}{2R + j\omega L} - \frac{1}{2R + \frac{1}{j\omega C}} = 0 \]

\[ 2R + \frac{1}{j\omega C} = 2R + j\omega L \]

\[ \frac{1}{j\omega C} = j\omega L \]

\[ 1 = -\omega^2 LC \]

\[ \omega^2 = -\frac{1}{LC}, \text{ which is not possible.} \]

Since, the value of \( L \) and \( C \) are positive number and square of angular frequency is always positive.

Hence, the correct option is (D).

**Given**: \( S_g = 1 \mu\text{A/mm}, R_g = 2.5 \text{k}\Omega \)

Resolution = 1 mm

For minimum change in \( R \) i.e., one scale

The current through the galvanometer is given by,

\[ I_g = S_g \times 1 \text{ mm} \]

\[ I_g = 1 \mu\text{A/mm} \times 1 \text{ mm} = 1 \mu\text{A} \]

The equivalent figure is represented as,

Applying KVL in loop \( CBAND \),

\[ 15 - 6 \times 10^3 (I_1) - 4 \times 10^3 (1 \times 10^{-6} + I_2) = 0 \]

\[ 15 - (6 \times 10^3 + 4 \times 10^3)I_2 - 4 \times 10^{-3} = 0 \]

\[ I_2 = \frac{15 - 0.004}{10 \times 10^3} = 1.4996 \text{ mA} \]

The value of \( V_{AN} \) is given by,

\[ V_{AN} = 4 \times 10^3 \times (I_2 + I_g) \]

\[ V_{AN} = 4 \times 10^3 \times (1.4996 \times 10^{-3} + 1 \times 10^{-6}) \]

\[ V_{AN} = 6.0024 \text{ V} \]

Hence, the correct option is (B).

**Given Wheatstone bridge is shown below,**

**Given**: \( R = 2 \times 10^3 \Omega + \Delta R \)

\( R_g = 2.5 \text{k}\Omega \)

\( V_{AN} = 6.0024 \text{ V} \)
Applying KVL in loop ABCDN,
\[ 15 = 6.0024 + (1 \times 10^{-6} \times 2.5 \times 10^3) + (I_1 \times 3 \times 10^3) \]
\[ I_1 = 2.9983 \times 10^{-3} \] ... (i)
Applying KVL in loop BDC,
\[ 15 - (3 \times 10^3 \times I_1) - (2 \times 10^3 + \Delta R)(I_1 - I_2) = 0 \]
From equation (i),
\[ 15 - (3 \times 10^3 \times 2.9983 \times 10^{-3}) - (2 \times 10^3 + \Delta R)(2.9983 \times 10^{-3} - 1 \times 10^{-6}) = 0 \]
\[ 6.0051 - 5.9946 - (\Delta R \times 2.9973 \times 10^{-3}) = 0 \]
\[ \Delta R = 3.5 \]
Hence, the correct option is (B).

### Method 1

Applying the bridge balance condition,
\[ \left( R + \frac{1}{j \omega C} \right) \left( \frac{R \times \frac{1}{j \omega C}}{R + \frac{1}{j \omega C}} \right) = RZ \]
\[ \left( 1 + \frac{j \omega RC}{j \omega C} \right) \times \frac{R}{(1 + j \omega RC)} = RZ \]
\[ Z = \frac{1}{j \omega C} \]
Thus, the impedance \( Z \) is a pure capacitor.
Hence, the correct option is (A).

### Method 2
The above bridge can be modified as,

Comparing the above both figure,
The element \( Z \) will be the pure capacitor.
Hence, the correct option is (A).

**Given**: \( R_i \) increases by 1 \( \Omega \) i.e.
\[ R_i = 1 \, \text{k}\Omega + 1 = 1001 \, \Omega \]

**Calculation of \( V_{th} \):**
From figure,
\[ V_{AB} = 10 \times \frac{1 \, \text{k}\Omega}{1 \, \text{k}\Omega + 1 \, \text{k}\Omega} = 5 \, \text{V} \] [By VDR]
\[ V_{AD} = 10 \times \frac{1001 \, \Omega}{1000 + 1001} = 5.002498 \]
\[ V_{th} = V_{AB} - V_{AD} \]
\[ V_{th} = 5.002498 - 5 = 0.002498 \]

**Calculation of Thevenin’s resistance :**

(When dependent sources are not present)
Replace all independent sources by their internal resistance i.e. short circuit all the independent voltage source \( R_{in} = 0 \) and open circuit all the independent current source \( R_{in} = \infty \).

\[ R_{th} = 1000.2498 \]
\[ R_{th} = 1000.2498 \]
\[ \text{Hence, the correct option is (A).} \]

\[ I_g = \frac{0.002498}{1000.2498 + 1000} = 1.25 \, \mu\text{A} \]

(i) Heaviside Campbell bridge is used for finding the mutual inductance.
(ii) Schering bridge is used for finding the self-capacitance and dissipation factor.
(iii) De-Sauty bridge is used for finding self capacitance.
(iv) Wien bridge is used for finding frequency.

Hence, the correct option is (A).

\[ 3.32 \text{ (B)} \]

**Condition for bridge balance :**
Product of opposite arm impedance should be equal i.e.
\[ Z_{AB}Z_{CD} = Z_{BC}Z_{AD} \]
\[ R_iZ_2 = (R_4 + j\omega L_4)R_3 \]
\[ Z_2 = \frac{R_iR_4}{R_i} \, j\omega \frac{R_iL_4}{R_i} = R + j\omega L \]

where, \( R = \frac{R_iR_4}{R_i} \) and \( L = \frac{R_iL_4}{R_i} \)

From the above expression, it is clear that impedance \( Z \) will be the series combination of \( R \) and \( L \).

Hence, the correct option is (B).
Given bridge circuit is shown below,

**Given**: Bridge is balance.
Since, the bridge is balance, so the current through galvanometer is zero i.e. open circuit.

**Conversion from Δ to Y**:  
\[
\begin{align*}
R_1 &= \frac{100 \times 200}{100 + 200 + 100} = 50 \Omega \\
R_2 &= \frac{200 \times 100}{200 + 100 + 100} = 50 \Omega \\
R_3 &= \frac{100 \times 100}{100 + 200 + 100} = 25 \Omega
\end{align*}
\]

The equivalent figure is represented as,

**Condition for bridge balance**:  
Product of opposite arm impedances should be equal i.e.  
\[Z_{AB}Z_{CD} = Z_{AD}Z_{BC}\]

From figure,  
\[150 \times R_x = 100 \times 50\]

\[R_x = \frac{100}{3} = 33.33 \Omega\]

Hence, the value of \(R_x\) is **33.33 Ω**.

**Kelvin double bridge is used for measurement of low resistance (less than 1 Ω).**

Low resistance is represented by four terminals as,

Hence, the correct option is (B).

**Method 1**

Given balance bridge circuit is shown below,
Given: \( R = 10^3 \, \Omega \) and \( C = 10^{-7} \, \text{F} \)

The above bridge is the standard Wien’s bridge.

The frequency (\( \omega_0 \)) is given by,

\[
\omega_0 = \frac{1}{\sqrt{R \times C \times R \times C}}
\]

\[
\omega_0 = \frac{1}{RC} = \frac{1}{10^3 \times 10^{-7}} = 10000 \, \text{rad/sec}
\]

Hence, the value of \( \omega_0 \) is \textbf{10000 rad/s}.

Method 2

Since, the bridge is balance.

**Condition for bridge balance:**

(i) Product of opposite arm impedances should be equal i.e.

\[ Z_{CD} Z_{AB} = Z_{AD} Z_{BC} \]  

...(i)

(ii) The detector current should be zero.

From equation (i),

\[
\left( R + \frac{1}{j\omega_0 C} \right) (R) = \left( \frac{2R \times R}{1 + j\omega_0 CR} \right)
\]

\[
(1 + j\omega_0 CR)(1 + j\omega_0 CR)(R) = 2R \times R \times j\omega_0 C
\]

\[
(1 + j\omega_0 CR)(1 + j\omega_0 CR) = j\omega_0 2RC
\]

Comparing real and imaginary term,

For real part, \( 1 - \omega_0^2 R C R C = 0 \)

\[
\omega_0 = \frac{1}{\sqrt{R \times C \times R \times C}}
\]

\[
\omega_0 = \frac{1}{\sqrt{R \times C \times R \times C}}
\]

Given Wheatstone bridge is shown below,

\[ \text{Resolution} = 0.1 \, \text{mV}, \, R = 1000 \, \Omega \]

**Method 1**

The above bridge is the example of quarter bridge.

From quarter bridge,

\[
V_0 = \frac{V}{4} \times \frac{\Delta R}{R}
\]

From the given data,

\[
0.1 \, \text{mV} = \frac{2 \times \Delta R}{4 \times 1000}
\]

\[
\Delta R = 0.2 \, \Omega
\]

The minimum value of \( \Delta R \) is \textbf{0.2 \, \Omega}.

**Method 2**

The above figure can be represented as,

From figure,

\[
V_B = 2 \times \frac{1000}{1000 + 1000} = 1 \, \text{V}
\]
\[
V_A = 2 \times \frac{1000 + \Delta R}{1000 + 1000 + \Delta R} \\
V_{AB} = V_A - V_B \\
V_{AB} = 2 \left[ \frac{1000 + \Delta R}{1000 + 1000 + \Delta R} - \frac{1000}{1000 + 1000} \right]
\]

Also, \( V_{AB} \) = Resolution = 0.1 mV.

\[
0.1 \text{ mV} = 2 \left[ \frac{1000 + \Delta R}{2000 + \Delta R} - \frac{1}{2} \right]
\]

\[
1.0001 = \frac{2000 + 2\Delta R}{2000 + \Delta R} \\
\Delta R = \frac{2000.2 - 2000}{2 - 1.0001} = 0.2 \Omega
\]

The minimum value of \( \Delta R \) is 0.2 \( \Omega \).

### Method 2: Conventional Method

Given bridge is balance,

Condition for bridge balance,

(i) Product of opposite arm should be equal i.e., \( Z_{AB}Z_{CD} = Z_{AD}Z_{BC} \)  

(ii) Potential at points \( B \) and \( D \) should be same (i.e. detector current cannot be zero).

From figure,

\[
Z_{AB} = \frac{R_1 \times \frac{1}{j\omega C_1}}{1 + \frac{1}{j\omega R_1 C_1}} \]

\[
Z_{AD} = R_1, \quad Z_{BC} = R_4, \quad Z_{CD} = R_4 + j\omega L_x
\]

From equation (i),

\[
\left( \frac{R_1}{1 + j\omega R_1 C_1} \right) (R_4 + j\omega L_x) = R_2 R_4 \\
R_1 R_4 + j\omega R_1 L_x = R_2 R_4 + j\omega R_2 R_4 R_1 C_1
\]

Equating imaginary part on both side,

\[
\omega R_1 L_x = \omega R_2 R_4 R_1 C_1 \\
L_x = R_2 R_4 C_1 \\
L_x = 22.2 \times 10^3 \times 10 \times 10^3 \times 1 \times 10^{-9} \\
L_x = 222 \text{ mH}
\]

Hence, the value of inductance \( (L_x) \) is 222 mH.

\[
\Delta \Delta \Delta \Delta
\]