GATE Objective & Numerical Type Solutions

Question 4 [Practice Book]  
A unity feedback control system has the open loop transfer function.

\[ G(s) = \frac{4(1+2s)}{s^2(s+2)} \]

If the input to the system is a unit ramp, the steady state error will be

(A) 0  (B) 0.5  (C) 2  (D) \( \infty \)

**Ans. (A)**

**Sol. Given**: \( G(s) = \frac{4(1+2s)}{s^2(s+2)} \), and \( r(t) = tu(t) \)

Taking Laplace transform of \( r(t) \), we get

\[ R(s) = \frac{1}{s^2} \]

Steady state error is given by,

\[ e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)} \]

\[ e_{ss} = \lim_{s \to 0} \frac{s \times 1}{s^2} \left( \frac{4(1+2s)}{s^2(s+2)} \right) = \lim_{s \to 0} \frac{1}{s^2} \left( \frac{4(1+2s)}{s^2(s+2)} \right) = 0 \]

Hence, the correct option is (A).

**Alternatively,**

Velocity error coefficient is given by,

\[ K_v = \lim_{s \to 0} sG(s)H(s) = \lim_{s \to 0} \frac{s \times 4(1+2s)}{s^2(s+2)} = \infty \]

Steady state error for ramp input is given by,

\[ e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0 \]

*For type - 2 system steady state error due to ramp input will be zero.*

Question 6 [Practice Book]  
A first order system and its response to a unit step input are shown in figure below, the system parameters \( a \) and \( K \) are respectively

\[ r(t) \quad K \quad s + a \quad c(t) \]

(A) 5, 10  (B) 10, 5  (C) 2, 10  (D) 10, 2
Ans. (A)

Sol. For first order system loop transfer function is \( \frac{C(s)}{R(s)} = \frac{K}{1 + s\tau} \) and its response to a unit step input are shown in figure below.

For the given transfer function \( \frac{C(s)}{R(s)} = \frac{a}{1 + \frac{s}{a}} \) and its response.

From the figure, time constant is 0.2 sec.

On comparing with above standard equation, we get, \( \tau = \frac{1}{a} \quad \Rightarrow \quad a = 5 \).

Using final value theorem steady state output can be written as,

\[
c_{ss} = \lim_{t \to \infty} c(t) = \lim_{s \to 0} s C(s)
\]

\[
c_{ss} = \lim_{s \to 0} s \frac{K}{s + a} R(s) = \lim_{s \to 0} s \frac{K}{s + a} \times \frac{1}{s} = \frac{K}{a}
\]

From figure, \( c_{ss} = 2 \)

\[
\frac{K}{a} = 2 \quad \Rightarrow \quad K = 10
\]

Hence, the correct option is (A).

**Question 7 [Practice Book]**

Block diagram model of a position control system is shown in figure.

(a) In absence of derivative feedback \( (K_i = 0) \), determine damping ratio of the system for amplifier gain \( K_A = 5 \). Also find the steady state error to unit ramp input.

(b) Find suitable values of the parameters \( K_A \) and \( K_i \) so that damping ratio of the system is increased to 0.7 without affecting the steady-state error as obtained in part (a).
**Sol.**  **Given:** The given block diagram of a positional control system is shown in figure below.

(a) **Given:** $K_t = 0$

\[ G(s) = \frac{5}{s(0.5s + 1)} = \frac{10}{s(s + 2)} \quad \text{and} \quad H(s) = 1 \]

Closed-loop transfer function for negative feedback is given by,

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{10}{1 + \frac{10}{s(s + 2)}} = \frac{10}{s^2 + 2s + 10} \]

Transfer function for second-order system with unit step input is given by,

\[ C(s) = \frac{\omega_n^2}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]

where, $\xi = \text{damping ratio}, \quad \omega_n = \text{natural angular frequency}$

On comparing equation(i) and (ii), we get

\[ \omega_n = \sqrt{10} \quad \text{rad/sec.} \quad \text{and} \quad 2\xi \omega_n = 2 \]

\[ \xi = \frac{1}{\sqrt{10}} \]

Steady state error is given by,

\[ e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)} \]

\[ e_n = \lim_{s \to 0} \frac{s \times \frac{1}{10}}{1 + \frac{10}{s(s + 2)}} = \lim_{s \to 0} \frac{1}{s + \frac{10}{s + 2}} = 0.2 \quad \text{Ans.} \]

(b) The open loop transfer function can be written as,

\[ \text{OLTF} = \frac{2K_A}{s^2 + 2s + 2sK_t} = \frac{2K_A}{s[s + (2 + 2K_t)K_A]} \]

The closed loop transfer function can be written as,

\[ \text{CLTF} = \frac{Y(s)}{R(s)} = \frac{2K_A}{s^2 + s + (2 + 2K_t) + 2K_A} \]

Transfer function for second-order system with unit step input is given by,

\[ \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]

where, $\xi = \text{damping ratio}, \quad \omega_n = \text{natural angular frequency}$
Velocity error coefficient is given by,

\[ K_v = \lim_{s \to 0} s G(s) H(s) \]

\[ K_v = \lim_{s \to 0} s \frac{2K_A}{s[2 + 2K_t]} = \frac{2K_A}{2 + 2K_t} \]

Steady state error is given by,

\[ e_{ss} = \frac{1}{K_v} \]

\[ 0.2 = \frac{2K_A}{2 + 2K_t} \]

\[ 0.2K_A = 1 + K_t \quad \tag{iii} \]

On comparing equation (i) and (ii), we get

\[ \omega_n = \sqrt{2K_A} \]

\[ 2 + 2K_t = 2\xi\omega_n \]

\[ \xi = \frac{2 + 2K_t}{2\omega_n} = \frac{1 + K_t}{\sqrt{2K_A}} \]

\[ 0.7 = \frac{1 + K_t}{\sqrt{2K_A}} \]

\[ 1 + K_t = 0.9899\sqrt{K_A} \approx \sqrt{K_A} \quad \tag{iv} \]

From equation (iii) and equation (iv), we get

\[ K_A = 25 \quad \text{and} \quad K_t = 4. \quad \text{Ans.} \]

**Question 8 [Practice Book]**

For what values of ‘a’ does the system shown in figure have a zero steady state error [i.e., \( \lim_{t \to \infty} E(t) \)] for a step input?

(A) \( a = 0 \)  
(B) \( a = 2 \)  
(C) \( a \geq 4 \)  
(D) No value of ‘a’

**Ans.** (A)

**Sol.** The given system is shown in figure.

DC gain of feedback \( H(s) \) is \( K_H \)

\[ K_H = \lim_{s \to 0} H(s) = \lim_{s \to 0} \frac{1}{s + 4} = \frac{1}{4} \]
Closed-loop transfer function is given by,

\[
M(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{s + 1}{1 + \frac{s^2 + 5s + a}{(s + 1)(s^2 + 5s + a)(s + 4)}}
\]

\[
M(s) = \frac{(s + 1)(s + 4)}{(s^2 + 5s + a)(s + 4)(s + 1)} = \frac{(s + 1)(s + 4)}{s^3 + 4s^2 + 5s^2 + 20s + as + 4a + s + 1}
\]

\[
M(s) = \frac{(s + 1)(s + 4)}{s^3 + 9s^2 + (21 + a)s + (4a + 1)}
\]

Steady state error is given by,

\[
e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} R(s) \left[ \frac{1}{K_H} - M(s) \right]
\]

\[
e_{ss} = \lim_{s \to 0} s \left[ 4 - \frac{(s + 1)(s + 4)}{s^3 + 9s^2 + (21 + a)s + (4a + 1)} \right] R(s)
\]

For unit step input \( R(s) = \frac{1}{s} \)

\[
e_{ss} = \lim_{s \to 0} s \left[ 4 - \frac{(s + 1)(s + 4)}{s^3 + 9s^2 + (21 + a)s + (4a + 1)} \right]
\]

\[
0 = 4 - \frac{4}{4a + 1}
\]

\[
4 = \frac{4}{4a + 1}
\]

\[
16a + 4 = 4
\]

\[
a = 0
\]

Hence, the correct option is (A).

**Question 9 [Practice Book] [GATE IN 1993 IIT-Bombay : 2 Marks]**

A unit step is applied at \( t = 0 \) to a first order system without time delay. The response has a value of 1.264 units at \( t = 10 \) min and 2 units at steady state. The transfer function of the system is

(A) \( \frac{2}{1 + 600s} \) \hspace{1cm} (B) \( \frac{2}{600 + s} \) \hspace{1cm} (C) \( \frac{1}{1 + 600s} \) \hspace{1cm} (D) \( \frac{1}{600 + s} \)

**Ans. (A)**

**Sol.** The first order system step response is given by,

\[
c(t) = k \left[ 1 - e^{-\frac{t}{\tau}} \right]
\]

At \( t = \infty \), \( 2 = k \left[ 1 - e^{-\infty} \right] \)

\[
k = 2
\]

At \( t = 10 \) min,

\[
1.264 = k \left[ 1 - e^{-\frac{10}{\tau}} \right]
\]

\[
\tau = 10 \text{ min} = 600 \text{ sec.}
\]

First order transfer function is given by,

\[
G(s) = \frac{k}{1 + s\tau} = \frac{2}{1 + 600s}
\]

**Ans.**
A unity feedback closed loop second order system has a transfer function \( \frac{81}{s^2 + 0.6s + 9} \) and is excited by a step input of 1 unit. The steady state error of the output is

(A) 10  
(B) 0.0  
(C) 1.0  
(D) 0.1

Ans. (B)

Sol. Given: \( T(s) = \frac{81}{s^2 + 0.6s + 9} = \frac{9}{s^2 + 0.6s} = \frac{KG(s)}{1 + G(s)} \) …..(i)

Closed-loop transfer function for negative feedback is given by,

\( T(s) = \frac{G(s)}{1 + G(s)} \) …..(ii)

On comparing equation (i) and (ii), we get

\( G(s) = \frac{9}{s^2 + 0.6s} \)

Steady state error is given by,

\[
e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}
\]

\[
= \frac{1}{1 + \frac{s^2}{9}} = 0
\]

Hence, the correct option is (B).

The figure shows the block diagram representation of a control system. The system in block A has an impulse response \( h_A(t) = e^{-t}u(t) \). The system in block B has an impulse response \( h_B(t) = e^{-2t}u(t) \). The block ‘k’ is an amplifier by a factor \( k \). For the overall system the input is \( x(t) \) and output \( y(t) \).

(a) Find the transfer function \( \frac{Y(s)}{X(s)} \) when \( k = 1 \).

(b) Find the impulse response when \( k = 0 \).

(c) Find the values of \( k \) for which the system becomes unstable.

Sol. Given: Impulse response \( h_A(t) = e^{-t}u(t) \)

Taking Laplace transform, we get

\( H_A(s) = \frac{1}{s + 1} \)

Impulse response \( h_B(t) = e^{-2t}u(t) \)

Taking Laplace transform, we get

\( H_B(s) = \frac{1}{s + 2} \)
(a) $k = 1$, \quad \quad G(s) = H_A(s) \cdot H_B(s)$

Closed-loop transfer function for negative feedback is given by,

$$T(s) = \frac{Y(s)}{X(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$T(s) = \frac{1}{(s+1)(s+2)} = \frac{1}{s^2 + 3s + 3}$$  \quad \text{Ans.}$$

(b) $k = 0$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$A = 1$ and $B = -1$

$$H(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

Taking inverse Laplace transform, we get

Impulse response $h(t) = (e^{-t} - e^{-2t}) u(t)$ \quad \text{Ans.}$

(c) Value of $k$ for unstable system

$$T(s) = \frac{k}{s^2 + 3s + 2 + k}$$

Routh Tabulation:

<table>
<thead>
<tr>
<th>$s^2$</th>
<th>1</th>
<th>2 + k</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s^1$</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$s^0$</td>
<td>2 + k</td>
<td></td>
</tr>
</tbody>
</table>

For instability $2 + k < 0$

$\therefore k < -2$ system is unstable. \quad \text{Ans.}$

**Question 8 [Work Book]**  \quad [GATE EC 1999 IIT-Bombay : 2 Marks]

If the closed-loop transfer function $T(s)$ of a unity negative feedback system is given by

$$T(s) = \frac{a_{n-1}s + a_n}{s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n}$$

then the steady state error for a unit ramp input is

(A) $\frac{a_n}{a_{n-1}}$  \quad (B) $\frac{a_n}{a_{n-2}}$  \quad (C) $\frac{a_{n-1}}{a_{n-2}}$ \quad (D) zero

Ans.  \text{(D)}

Sol. \quad \text{Given : } T(s) = \frac{a_{n-1}s + a_n}{s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n} \quad \text{and } r(t) = tu(t)

\begin{align*}
T(s) &= \frac{\frac{a_{n-1}s + a_n}{s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n}}{1 + \frac{a_{n-1}s + a_n}{s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n}} \\
&= \frac{a_{n-1}s + a_n}{s^n + a_1s^{n-1} + \ldots + a_{n-1}s + a_n} \quad \text{...... (i)}
\end{align*}
Closed-loop transfer function for negative feedback is given by,

\[ T(s) = \frac{G(s)}{1 + G(s)H(s)} \] .... (ii)

On comparing equation (i) and (ii), we get

\[ G(s) = \frac{a_{n-1}s + a_n}{s^n + a_1s^{n-1} + \ldots + a_{n-2}s^2} \]

Taking Laplace transform of \( r(t) \), we get

\[ R(s) = \frac{1}{s^2} \]

The steady state error due to a unit ramp input is given by,

\[ e_{ss} = \frac{1}{K_v} \]

Velocity error coefficient is given by,

\[ K_v = \lim_{s \to 0} sG(s)H(s) = \frac{s(a_{n-1}s + a_n)}{s^n + a_1s^{n-1} + \ldots + a_{n-2}s^2} = \infty \]

So, that \[ e_{ss} = \frac{1}{K_v} = \frac{1}{\infty} = 0 \]

Hence, the correct option is (D).

**Question 31 [Practice Book] [GATE EC 2000 IIT-Kharagpur : 5 Marks]**

The block diagram of a feedback system is shown in the figure.

(a) Find the closed loop transfer function.
(b) Find the minimum value of \( G \) for which the step response of the system would exhibit an overshoot, as shown in figure.
(c) For \( G \) equal to twice the minimum value, find the time period \( T \) indicated in the figure.

**Sol.**
(a) The given block diagram of a feedback system is shown below.
Closed-loop transfer function for negative feedback is given by,

\[ T(s) = \frac{G(s)}{1 + G(s)} = \frac{G}{s(s + 3)} \]

\[ T(s) = \frac{G}{s^2 + 3s + G} \]

\[ G \]

\[ T(s) = \frac{G}{s(s + 3) + G} \]

(b) **Given**: Maximum peak overshoot is given by,

\[ \text{MPO} = e^{\sqrt{-1} \xi} = 0.1 \]

\[ \therefore \text{Damping ratio is } \xi = 0.6 \]

For the given response transfer function is

\[ T(s) = \frac{G}{s^2 + 3s + G} \]

Transfer function for second-order system is given by,

\[ C(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]

\[ R(s) = \frac{s^2 + 3s + G}{G} \]

where, \( \xi = \text{damping ratio}, \quad \omega_n = \text{natural angular frequency} \)

On comparing equation (i) and equation (ii), we get

\[ 2\xi \omega_n = 3 \quad \text{and} \quad \omega_n^2 = G \]

\[ \omega_n = \frac{3}{2\xi} = \sqrt{G} \]

\[ \sqrt{G} = \frac{3}{2 \times 0.6} \]

\[ G = 6.25 \]

(c) \( G' = 2G = 2 \times 6.25 = 12.5 \)

\[ \omega_n^2 = G' \quad \Rightarrow \quad \omega_n = \sqrt{12.5} = 3.53 \]

\[ 2\xi \omega_n = 3 \quad \Rightarrow \quad \xi = 0.424 \]

Damped frequency of oscillation is given by,

\[ \omega_d = \omega_n \sqrt{1 - \xi^2} \]

\[ \frac{2\pi}{T} = 3.53 \sqrt{1 - (0.424)^2} = 3.197 \]

\[ T = 1.96 \text{ sec.} \]

**Question 37 [Practice Book] [GATE IN 2002 IISc-Bangalore : 2 Marks]**

The forward path transfer function of an unity feedback system is given by,

\[ G(s) = \frac{(1 + 5s)(1 + 10s)(1 + 2s)}{(1 + s)(1 + 8s)(1 + 20s)} \]

If \( e(t) \) is the error to a unit impulse input the value of the performance index \( J = \int_0^\infty e(t) dt \) is equal to

(A) zero \quad (B) infinity \quad (C) \(-12\) \quad (D) 0.5

\[ \text{Ans. (D)} \]
Given : \( r(t) = \delta(t) \), \( R(s) = 1 \), \( H(s) = 1 \)

Therefore,

\[
E(s) = \frac{1}{1 + \frac{(1 + 5s)(1 + 10s)(1 + 2s)}{(1 + s)(1 + 8s)(1 + 20s)}}
\]

\[
E(s) = \frac{(1 + s)(1 + 8s)(1 + 20s)}{(1 + s)(1 + 8s)(1 + 20s) + (1 + 5s)(1 + 10s)(1 + 2s)}
\]

For causal \( e(t) \):

\[
E(s) = \int_{0}^{\infty} e(t)e^{-st}dt
\]

Given \( J = \int_{0}^{\infty} e(t) dt \)

\[
J = E(s)\bigg|_{s=0}
\]

\[
J = \lim_{s \to 0} \frac{(1 + s)(1 + 8s)(1 + 20s)}{(1 + s)(1 + 8s)(1 + 20s) + (1 + 5s)(1 + 10s)(1 + 2s)}
\]

\[
J = \frac{1 \times 1 \times 1}{(1 \times 1 \times 1) + (1 \times 1 \times 1)} = 0.5
\]

**Ans.**

**Question 11 [Work Book] [GATE EC 2003 IIT-Madras : 2 Marks]**

A second-order system has the transfer function \( \frac{C(s)}{R(s)} = \frac{4}{s^2 + 4s + 4} \) with \( r(t) \) as the unit-step function, the response \( c(t) \) of the system is represented by

<table>
<thead>
<tr>
<th>(A) Step Response</th>
<th>(B) Step Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude</td>
<td>Amplitude</td>
</tr>
<tr>
<td>1.5</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td></td>
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<tr>
<td>0.0</td>
<td></td>
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<tr>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
Ans. (B)

Sol. Given : \[
\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4s + 4} 
\]  
Transfer function for second-order system is given by, \[
\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} 
\]  
where, \(\xi\) = damping ratio, \(\omega_n\) = natural angular frequency
\[
\omega_n^2 = 4 \quad \Rightarrow \quad \omega_n = 2 \text{ rad/sec}
\]
\[
2\xi\omega_n = 4 \quad \Rightarrow \quad \omega_n = 2 \text{ rad/sec}
\]
\[
\xi = 1
\]
Since \(\xi = 1\), system is critically damped.
The final value can be calculated using final value theorem,
\[
c_n = \lim_{s \to 0} s \cdot \frac{4R(s)}{s^2 + 4s + 4} = 1
\]
For 2% tolerance band settling time is given by,
\[
T_s = \frac{4}{\xi\omega_n} = \frac{4}{2} = 2 \text{ sec}
\]
This mean that the response \(c(t)\) will be settle to its final value after 2 sec.
Hence, the correct option is (B).

Question 13 [Work Book] [GATE IN 2004 IIT-Delhi : 2 Marks]

A certain system exhibited an overshoot of 16% when subjected to an input of \(2u(t)\), where \(u(t)\) is a step input. The damping ratio and decay ratio respectively are
(A) (0.8, 0.0810)  
(B) (0.5, 0.0256)  
(C) (1.0, 0.1626)  
(D) (1.1, 0.0089)

Ans. (B)

Sol. Given : MPO = 16%  [By default 1st peak overshoot]
Percentage MPO is given by,
\[
\% \text{ MPO} = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100
\]
\[
16 = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \times 100
\]
\[
-\frac{\pi\xi}{\sqrt{1-\xi^2}} = \ln(0.16)
\]
\[ \xi^2 = \frac{1}{3.939} \]
\[ \xi = 0.5 \]

Damping ratio \((\xi) = 0.5\)

**Concept of decay ratio:**

![Decay Ratio Diagram](image)

The 2\textsuperscript{nd} peak overshoot is given by,

\[ \%\text{MPO}(2\text{nd}) = e^{-\frac{3\pi\xi}{\sqrt{1-\xi^2}}} \times 100 \]

The decay ratio is given by,

\[ \text{Decay ratio} = \frac{e^{-\frac{3\pi\xi}{\sqrt{1-\xi^2}}}}{e^{-\frac{3\pi\xi}{\sqrt{1-(0.5)^2}}}} = \frac{e^{-\frac{3\pi(0.5)}{\sqrt{1-(0.5)^2}}}}{e^{-\frac{3\pi}{\sqrt{1-(0.5)^2}}}} = 0.00433 \approx 0.027 \]

Hence, the correct option is (B).

**Question 51 [Practice Book]**

RLC circuit shown in figure. For a step input \(e_i\), the overshoot in the output \(e_o\) will be

- (A) 0 %, since the system is not under damped
- (B) 5 %
- (C) 16 %
- (D) 48 %

**Ans.** (C)
The given RLC circuit is shown below.

Transform domain:

Characteristic equation is given by,

\[ s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \]  \(\text{...}(i)\)

Standard characteristic equation for second order system is given by,

\[ s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \]  \(\text{...}(ii)\)

On comparing equation (i) and equation (ii),

\[ \omega_n = \frac{1}{\sqrt{LC}} \quad \text{and} \quad 2\xi\omega_n = \frac{R}{L} \]

\[ \xi = \frac{R}{L} \times \frac{1}{2\omega_n} = \frac{R}{L} \times \frac{1}{2} \times \frac{1}{\sqrt{LC}} = \frac{R}{2L} \times \sqrt{\frac{C}{L}} \]

\[ \xi = \frac{10}{2} \sqrt{\frac{10 \times 10^{-6}}{1 \times 10^{-3}}} = 0.5 \]

Maximum peak overshoot is given by,

\[ \text{MPO} = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} = e^{-\pi \times 0.5} \]

\[ \text{MPO} = 0.163 \text{ or } 16.3\% \approx 16\% \]

Hence, the correct option is (C).
If the above step response is to be observed on a non-storage CRO, then it would be best have the $\xi$ as a

(A) step function   (B) square wave of 50 Hz  
(C) square wave of 300 Hz    (D) square wave of 2 kHz

Ans.  (C)

Sol.  The resonance frequency is given by,

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-3}\times10\times10^{-6}}}$$

$$\omega_n = 10^4 \text{ rad/sec}$$

Settling time for 2% tolerance band is given by,

$$t_s = \frac{4}{\xi \omega_n} = \frac{4}{0.5\times10^4} = 0.8 \text{ msec.}$$

To observe the transient response, input must be applied at least up to $t_s = 0.8 \text{ msec}$. 

Peak time is given by,

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \xi^2}} = \frac{\pi}{10^4 \sqrt{1 - (0.5)^2}} = 0.36 \text{ msec}$$

To observe peak overshoot, the input must be applied at least up to $t_p = 0.36 \text{ msec}$. 

**Case 1 :** If step input is applied.

The given CRO is non-storage which means it can’t record the response. So it is not possible to read the transient response practically as it will appear only for one $t_s$ i.e., 0.8 msec. After 0.8 msec CRO only displays constant value 1.

Therefore, the constant input $e_1 = 1$ should be applied again and again (i.e., square wave) so that transient response appears always on CRO screen.

**Case 2 :** If square wave input is applied.
For obtaining transient response, 
\[ \frac{T}{2} > (t_p \text{ and } t_s) \]

**Option (B):**
For \( f_1 = 50 \text{ Hz} \)

\[ \frac{T_1}{2} = \frac{1}{2 \times 50} = 10 \text{ ms} \gg t_s \]

10 ms \( \gg \gg \) 0.8 ms

The output of the CRO is constant in between 0.8 ms to 10 ms as shown below,

Option (C): 
For \( f_2 = 300 \text{ Hz} \)

\[ \frac{T_2}{2} = \frac{1}{2 \times 300} = 1.67 \text{ ms} \gg t_s \]

The output of the CRO is constant in between 0.8 ms to 1.67 ms which is very less as shown below,

Option (D): 
For \( f_3 = 2 \text{ kHz} \)

\[ \frac{T_3}{2} = \frac{1}{2 \times 2 \times 10^3} = 0.25 \text{ ms} \ll t_s \]

Since, 0.25 ms is very much less than 0.8 ms hence, the output waveform will not appear on the CRO screen.

Hence 300 Hz frequency square wave is the most suitable as \( \frac{T}{2} = 1.67 \text{ msec} \) because only for this frequency transient response will always appear on CRO screen.

Therefore, it would be best to have \( e_i \) as a square wave of 300 Hz.

Hence, the correct option is (C).

**Question 53 [Practice Book]**

Consider the feedback system shown below which is subjected to a unit step input. The system is stable and as following parameters \( K_p = 4, K_i = 10, \omega = 500 \text{ rad/sec and } \xi = 0.7 \). The steady state value of \( Z \) is.
Ans. (A)

Sol. Given: \( R(s) = \frac{1}{s} \)

The given feedback system is shown below.

\[
G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}
\]

The equivalent representation can be drawn as shown in figure below.

Closed-loop transfer function for negative unity feedback is given by,

\[
T(s) = \frac{C(s)}{R(s)} = \frac{G'(s)}{1 + G'(s)}
\]

\[
\frac{C(s)}{R(s)} = \frac{\left( K_p + \frac{K_i}{s} \right) G(s)}{1 + \left( K_p + \frac{K_i}{s} \right) G(s)}
\]

From the figure, the error signal can be written as,

\[
E(s) = R(s) - C(s) = R(s) \left[ 1 - \frac{\left( K_p + \frac{K_i}{s} \right) G(s)}{1 + \left( K_p + \frac{K_i}{s} \right) G(s)} \right]
\]

\[
E(s) = \frac{1}{s} \left[ \frac{1}{1 + \left( K_p + \frac{K_i}{s} \right) G(s)} \right]
\]

From the figure,

\[
Z(s) = \frac{K_i}{s} E(s) = \frac{K_i}{s^2} \left[ \frac{s}{s + (sK_p + K_i) G(s)} \right]
\]
Steady state value can be calculated as,

\[ z_{ss} = \lim_{t \to \infty} z(t) = \lim_{s \to 0} s Z(s) = K_i \times \frac{\omega_n^2}{\omega_n^2 \times K_i} = 1 \]

Hence, the correct option is (A).

**Question 63 [Practice Book] [GATE EE 2011 IIT-Madras : 2 Marks]**

A two-loop position control system is shown below.

The gain \( k \) of the Tacho-generator influences mainly by

(A) peak overshoot.
(B) natural frequency of oscillation.
(C) phase shift of the closed loop transfer function at very low frequency \( (\omega \to 0) \).
(D) phase shift of the closed loop transfer function at very high frequency \( (\omega \to \infty) \).

**Ans.** (A)

**Sol.** Given: A two-loop position control system is shown below.

On solving the inner loop, we get

\[
\frac{1}{s(s+1)} \times \frac{ks}{1+\frac{ks}{s(s+1)}} = \frac{1}{s^2 + s(1+k)}
\]

Now, the overall transfer function can be written as,

\[
\frac{Y(s)}{R(s)} = \frac{1}{s^2 + (k+1)s + 1} \quad \text{.....(i)}
\]
Transfer function for second-order system with unit step input is given by,

\[ \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \]  

.....(ii)

where, \( \xi = \) damping ratio, \( \omega_n = \) natural angular frequency

On comparing equation (i) and equation (ii), we get \( \omega_n = 1 \) and \( 2\xi\omega_n = k + 1 \)

So \( \xi = \frac{k + 1}{2} \)  

.....(iii)

Maximum peak overshoot is given by,

\[ MPO = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \quad 0 \leq MPO \leq 1, \quad 0 \leq \xi \leq 1 \]

Peak overshoots depends on damping factor \( \xi \) and \( \xi \) is proportional to gain from equation (iii). So, gain \( k \) of the Tacho-generator influences mainly by peak overshoot.

Hence, the correct option is (A).

**Question 20 [Work Book]**  

The open-loop transfer function of a dc motor is given as \( \frac{\omega(s)}{V_a(s)} = \frac{10}{1 + 10s} \). When connected in feedback as shown below, the approximate value of \( K_a \) that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is

(A) 1  
(B) 5  
(C) 10  
(D) 100

**Ans.** (C)

**Sol.** Given: \( \tau_{\text{closed loop}} = \frac{1}{100} \tau_{\text{open loop}} \) where \( \tau \) represents time constant and \( \frac{\omega(s)}{V_a(s)} = \frac{10}{1 + 10s} \)

For first order system loop transfer function is \( \frac{C(s)}{R(s)} = \frac{K}{1 + s\tau} \) comparing with \( \frac{\omega(s)}{V_a(s)} = \frac{10}{1 + 10s} \) \( \tau_{\text{open loop}} = 10 \)

Closed-loop transfer function for negative unity feedback is given by,

\[ T(s) = \frac{G(s)}{1 + G(s)} \]

Here \( G(s) = K_a \left( \frac{10}{1 + 10s} \right) \)

\[ \frac{\omega(s)}{R(s)} = \frac{K_a \left( \frac{10}{1 + 10s} \right)}{1 + K_a \left( \frac{10}{1 + 10s} \right)} = \frac{K_a 10}{10s + K_a 10} = \frac{10K_a}{10s + (10K_a + 1)} \]
Dividing numerator and denominator by $10K_a + 1$

$$\frac{\omega(s)}{R(s)} = \frac{\frac{10K_a}{10K_a + 1}}{1 + \left(\frac{10}{10K_a + 1}\right)s}$$

For first order system loop transfer function is $\frac{C(s)}{R(s)} = \frac{K}{1 + s\tau}$. On comparing with $\frac{\omega(s)}{R(s)}$ we get

$$\tau_{\text{closed loop}} = \frac{10}{10K_a + 1}$$

We have

$$\tau_{\text{closed loop}} = \frac{1}{100} \tau_{\text{open loop}}$$

$$\frac{10}{10K_a + 1} = \frac{1}{100} 10$$

$$10K_a + 1 = 100$$

$$10K_a = 99$$

$$K_a = 9.9 = 10$$

Hence, the correct option is (C).

**Question 67 [Practice Book]**

The position control of a DC servo-motor is given in the figure. The values of the parameters are $K_T = 1$ N-m/A, $R_a = 1\Omega$, $L_a = 0.1$H, $J = 5$ kg-m$^2$, $B = 1$ N-m/(rad/sec) and $K_h = 1$ V/(rad/sec).

The steady-state position response (in radians) due to unit impulse disturbance torque $T_d$ is _______.

**Ans.** – 0.5

**Sol.** Given: $K_T = 1$ N-m/A, $R_a = 1\Omega$, $L_a = 0.1$H, $J = 5$ kg-m$^2$, $B = 1$ N-m/(rad/sec), $K_h = 1$ V/(rad/sec)

For unit impulse $T_d(s) = 1$

$$\frac{X(s)}{-T_d(s)} = \frac{1}{Js + B} \frac{1}{1 + \frac{1}{(Js + B)(R_a + L_a s)}} K_s K_T$$
Steady state response can be calculated using final value theorem. Applying final value theorem,
\[ \theta(0) = \lim_{s \to 0} s \cdot \theta(s) = \lim_{s \to 0} \frac{-1}{s[(Js + B)(R_a + L_a s) + K_b K_T]} \]
\[ \theta(0) = \frac{-1}{BR_a + K_b K_T} = \frac{-1}{1+1} = -0.5 \]
Hence, the correct answer is – 0.5.

**Question 68 [Practice Book]**

In the feedback system shown below, \( G(s) = \frac{1}{(s^2 + 2s)} \).
The step response of the closed-loop system should have minimum settling time and have no overshoot.

The required value of gain \( K \) to achieve this is ________.

**Ans. 1**

**Sol.**

Given: \( G(s) = \frac{1}{s^2 + 2s}, \ G'(s) = KG(s) \)

The second order closed loop transfer function with negative unity feedback is given by,
\[ \frac{Y(s)}{R(s)} = \frac{G'(s)}{1 + G'(s)} = \frac{K}{s^2 + 2s} = \frac{K}{s^2 + 2s + K} \quad ...... (i) \]

Minimum settling time and no overshoot
\( \xi = 1 \)
From equation (i),
\[ \omega_n = \sqrt{K} \]
And
\[ 2\xi \omega_n = 2 \]
\[ 2\xi \sqrt{K} = 2 \]
\[ \xi = \frac{1}{\sqrt{K}} = 1 \]
\[ \sqrt{K} = 1 \]
\[ K = 1 \]
IES Objective Solutions

Question 4 [Practice Book] [IES EE 1992]

Damping factor and un-damped natural frequency for the position control system is given by

(A) \( 2\sqrt{KJ}, \sqrt{KJ} \) respectively
(B) \( \frac{K}{2fJ}, \frac{\sqrt{K}}{J} \) respectively
(C) \( \frac{f}{2\sqrt{KJ}}, \frac{\sqrt{K}}{J} \) respectively
(D) \( \frac{J}{2\sqrt{Kf}}, \sqrt{KJ} \) respectively

Ans. (C)
Sol. Characteristic equation of position control system is given by,
\[ s^2 + \frac{f}{J}s + \frac{K}{J} = 0 \] ....(i)

Standard form of second order characteristic equation is given by,
\[ s^2 + 2\xi \omega_n s + \omega_n^2 = 0 \] ....(ii)

On comparing equation (i) and (ii), we get
\[ 2\xi \omega_n = \frac{f}{J} \quad \omega_n = \frac{K}{J} \]
\[ \xi = \frac{f}{2\sqrt{Kf}} \quad \omega_n = \frac{\sqrt{K}}{J} \text{ rad/sec} \]

Hence, the correct option is (C).

Question 14 [Practice Book] [IES EE 1995]

Consider a system shown in the given figure.

If the system is distributed so that \( c(0) = 1 \), then \( c(t) \) for a unit step input will be

(A) \( 1 + t \)  
(B) \( 1 - t \)  
(C) \( 1 + 2t \)  
(D) \( 1 - 2t \)

Ans. (C)
Sol. Given :
\[ \frac{C(s)}{U(s)} = \frac{2}{s} \]
\[ C(s) = \frac{2}{s} U(s) \]
\[ C(s) = \frac{2}{s^2} \]

Taking inverse Laplace transform, we get
\[ c(t) = L^{-1}\left[\frac{2}{s^2}\right] = 2t \]
\[ c(t) = 2t + c(0) = 2t + 1 \]

Hence, the correct option is (C).
The unit impulse response of a system having transfer function \( \frac{K}{s + \alpha} \) is shown above. The value of \( \alpha \) is:

(A) \( t_1 \)  
(B) \( \frac{1}{t_1} \)  
(C) \( t_2 \)  
(D) \( \frac{1}{t_2} \)

Ans. (D)

Sol. Given:

\[
\frac{C(s)}{R(s)} = \frac{K}{s + \alpha} \quad \text{and} \quad R(s) = 1
\]

\[
C(s) = \frac{K}{s + \alpha}
\]

Taking inverse Laplace transform, we get

\[
c(t) = Ke^{-\alpha t}
\]

Time constant \( t = \frac{1}{\alpha} \)

Time constant is the time at which

\[
c(t) = Ke^{-1} = 0.37K
\]

So,

\[
\tau = t_2 = \frac{1}{\alpha}
\]

\[
\alpha = \frac{1}{t_2}
\]

Hence, the correct option is (D).

Question 54 [Practice Book]

Which one of the following is the steady state error of a control system with step error, ramp error and parabolic error constants \( K_p, K_v \) and \( K_a \) respectively for the input \( (1-t^2)u(t) \)?

(A) \( \frac{3}{1-K_p} - \frac{3}{2K_a} \)  
(B) \( \frac{3}{1+K_p} + \frac{3}{K_a} \)  
(C) \( \frac{3}{1+K_p} - \frac{3}{K_a} \)  
(D) \( \frac{3}{1+K_p} - \frac{6}{K_a} \)

Ans. (D)

Sol. Given:

\( r(t) = 3(1-t^2)u(t) \)

Taking Laplace transform, we get

\[
R(s) = \frac{3}{s} - \frac{6}{s^3}
\]
Steady state error is given by,

\[ e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)} = \lim_{s \to 0} \frac{s \left( \frac{3}{s} - \frac{6}{s^2} \right)}{1 + G(s)} \]

\[ e_{ss} = \lim_{s \to 0} \frac{3}{1 + G(s)} - \lim_{s \to 0} \frac{6}{s^2 + s^2 G(s)} \]

\[ e_{ss} = \frac{3}{1 + \lim_{s \to 0} G(s)} - \lim_{s \to 0} \frac{6}{s^2 G(s)} \]

\[ e_{ss} = \frac{3}{1 + K_p} - \frac{6}{K_a} \]

Hence, the correct option is (D).

**Question 72 [Practice Book]**

For a unity feedback control system with forward path transfer function \( G(s) = \frac{K}{s + 5} \), what is error transfer function \( W_e(s) \) used for determination of error coefficients?

(A) \( \frac{K}{s + 5} \)

(B) \( \frac{K}{s + K + 5} \)

(C) \( \frac{s + 5}{s + K + 5} \)

(D) \( \frac{K(s + 5)}{s + K + 5} \)

**Ans.** (C)

**Sol.** Given : \( G(s) = \frac{K}{s + 5} \), \( H(s) = 1 \)

\[ W_e(s) = \frac{1}{R(s) \left( 1 + G(s) \right)} \]

\[ W_e(s) = \frac{1}{R(s) \left( 1 + \frac{K}{s + 5} \right)} \]

\[ W_e(s) = \frac{s + 5}{R(s) \left( s + 5 + K \right)} \]

Hence, the correct option is (C).

**Question 77 [Practice Book]**

In the time domain analysis of feedback control systems which one pair of the following is not correctly matched?

(A) Under damped : Minimize the effect of nonlinearities

(B) Dominant poles : Transients die out more rapidly

(C) Far away poles to the left half of \( s \)-plane : Transients die out more rapidly

(D) A pole near to the left of dominant complex poles and near a zero : Magnitude of transient is small

**Ans.** (B)

**Sol.** Dominant Pole : The poles that are close to the imaginary axis in the left-half \( s \)-plane give rise to transient responses that will decay relatively slowly, whereas the poles that are far away from the axis (relative to the dominant poles) correspond to fast-decaying time responses.

Hence, the correct option is (B).
Question 89 [Practice Book] [IES EE 2009]

In a fluid flow system two fluids are mixed in appropriate proportion. The concentration at the mixing point is \( y(t) \) and it is reproduced without change, \( T_d \) seconds later at the monitoring point as \( b(t) \). What is the transfer function between \( b(t) \) and \( y(t) \)? (Where \( S \) is distance between monitoring point and mixing point)

(A) \( e^{-T_d s} \)  
(B) \( e^{+T_d s} \)  
(C) \( e^{-T_d s} \)  
(D) \( e^{+T_d s} \)  

Ans. (C)  

Sol. Given : \( y(t) = b(t - T_d) \)  

Taking Laplace transform, we get  
\[
Y(s) = e^{-T_d s}B(s)  
\]
\[
\frac{Y(s)}{B(s)} = e^{-T_d s}  
\]

Hence, the correct option is (C).

Question 97 [Practice Book] [IES EC 2011, 2001]

When two identical first order systems have been cascaded non-interactively the unit step response on the system will be

(A) Over-damped  
(B) Under-damped  
(C) Un-damped  
(D) Critically-damped  

Ans. (D)  

Sol. Cascading of two first order system with non-interactively

\[ T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{1 + sRC} \]

The pole-zero diagram of above transfer function

Hence, the correct option is (D).

Question 108 [Practice Book] [IES EC 2012]

Assertion (A) : A second order system subjected to a unit impulse oscillates at its natural frequency.  
Reason (R) : Impulse input contains frequencies from \(-\infty\) to \(+\infty\).  
Codes :  
(A) Both A and R are individually true and R is the correct explanation of A.
(B) Both A and R are individually true but R is not the correct explanation of A.
(C) A is true but R is false.
(D) A is false but R is true.

Ans. (D)

Sol. For \( \xi = 0 \), a second order system subjected to a unit impulse oscillates at its natural frequency but it is not always true.

\[
\begin{array}{c|c|c|c}
\xi = 0 & \text{Un-damped} & c(t) & \text{Marginal stable} \\
\hline
& & & \\
\end{array}
\]

The impulse response contains all the frequency components having frequency response as

\[
\mathcal{L}[\delta(t)]
\]

Hence, the correct option is (D).

**Question 131 [Practice Book] [IES EE 2014]**

The dominant poles of a servo-system are located at \( s = (-2 \pm j 2) \). The damping ratio of the system is

(A) 1  
(B) 0.8  
(C) 0.707  
(D) 0.6

Ans. (C)

Sol. Given: Pole are located at \( s = -2 \pm 2j \) .... (i)

Poles of second-order transfer function is given by,

\[
s = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}
\]

.... (ii)

On comparing equation (i) and (ii), we get

\[
\omega_n \sqrt{1 - \xi^2} = 2
\]

\[
\omega_n^2 (1 - \xi^2) = 4 \Rightarrow \omega_n^2 - \omega_n^2 \xi^2 = 4
\]

.... (iii)

and

\[
\xi \omega_n = 2
\]

\[
\xi \omega_n^2 = 4
\]

.... (iv)

From equation (iii) and (iv), we get

\[
\omega_n^2 = 8
\]

\[
\xi^2 = \frac{4}{8} = \frac{1}{2}
\]

\[
\xi = 0.707
\]

Hence, the correct option is (C).
A unity feedback second order control system is characterized by the open loop transfer function

\[ G(s) = \frac{K}{s(Js + B)}, \; H(s) = 1 \]

\( J = \) moment of inertia, \( B = \) damping constant and \( K = \) system gain

The transient response specification which is not affected by system gain variation is

(A) Peak overshoot  (B) Rise time  (C) Settling time  (D) Time to peak overshoot

Ans. (C)

Sol. Given: \( G(s) = \frac{K}{s(Js + B)} \) and \( H(s) = 1 \)

Characteristics equation is given by,

\[ 1 + G(s)H(s) = 0 \]

\[ Js^2 + Bs + K = 0 \Rightarrow s^2 + \frac{Bs}{J} + \frac{K}{J} = 0 \]  \( \text{.... (i)} \)

Characteristics equation for standard second-order system is given by,

\[ s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \]  \( \text{.... (ii)} \)

On comparing equation (i) and (ii), we get

\[ \omega_n = \sqrt{\frac{K}{J}} \text{ and } 2\xi\omega_n = \frac{B}{J} \]

\[ \xi = \frac{B}{2\sqrt{KJ}} \]

Peak overshoot is given by,

\[ M_p = e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}} \]

Since \( \xi \) depends on gain \( K \) so peak overshoot affected by \( K \).

Rise time is given by,

\[ t_r = \frac{\pi - \theta}{\omega_d} \]

Sine \( \omega_d \) depends on gain \( K \) so rise time affected by \( K \), settling time is given by,

\[ t_s = \frac{4}{\xi\omega_n} = \frac{4}{\frac{B}{2\sqrt{KJ}}} = \frac{8J}{B} \]

Settling time is independent of gain \( K \) so \( t_s \) will not be affected by \( K \).

Hence, the correct option is (C).