## GATE Objective & Numerical Type Questions

### Question 1 [Practice Book]  
**[GATE IN 1992, IIT-Delhi : 2 Marks]**

A system has a complex pole pair of \((-1 \pm j/2)\) and a real zero of \((-3)\). The steady state output to a unit step input is 2. The transfer function of the system is _______________.

**Sol.**

Given: Poles are at \(s = -1 \pm j/2\) and zero is at \(s = -3\)

Transfer function can be written as,

\[
\frac{C(s)}{R(s)} = G(s) = \frac{K(s + 3)}{(s + 1)^2 + 2^2}
\]

For unit step input,

\[
r(t) = u(t), \quad R(s) = \frac{1}{s}
\]

Steady state value can be calculated using final value theorem.

Applying final value theorem,

\[
c_{ss} = \lim_{s \to 0} s C(s) = \frac{3K}{5} = 2
\]

**K** = \(\frac{10}{3}\)

\[
G(s) = \frac{10}{3} \left( \frac{s + 3}{s^2 + 2s + 5} \right)
\]

### Question 12 [Practice Book]  
**[GATE IN 1999 IIT-Bombay : 1 Mark]**

A transfer function has two zeros at infinity. Then the relation between the numerator degree \((N)\) and the denominator degree \((M)\) of the transfer function is

(A) \(N = M + 2\)    (B) \(N = M - 2\)    (C) \(N = M + 1\)    (D) \(N = M - 1\)

**Ans. (B)**

**Sol.** For two zeros to exist at \(s = \infty\). It indicate there are two extra poles compare to zero. Therefore, the numerator degree, \(N\) should be less than the denominator degree, \(M\) by 2 i.e. \(N = M - 2\)

Hence, the correct option is (B).

### Question 16 [Practice Book]  
**[GATE EE 2002 IISc-Bangalore : 2 Marks]**

A single input single output system with \(y\) as output and \(u\) as input, is describe by

\[
\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} + 10y = 5 \frac{du}{dt} - 3u
\]

For an input \(u(t)\) with zero initial conditions the above system produces the same output as with no input and with initial conditions \(\frac{dy}{dt}y(0^-) = -4, \quad y(0^-) = 1\)

Input \(u(t)\) is

(A) \(\frac{1}{5} \delta(t) - \frac{7}{25} e^{(3/5)y}u(t)\)    (B) \(\frac{1}{5} \delta(t) - \frac{7}{25} e^{-2y}u(t)\)

(C) \(-\frac{7}{25} e^{-(3/5)y}u(t)\)    (D) None of these

**Ans. (A)**
### Basics of Control Systems

**Question 17**

A control system is defined by the following mathematical relationship

\[
\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 12(1 - e^{-2t})
\]

The response of the system as \( t \to \infty \) is

(A) \( x = 6 \)

(B) \( x = 2 \)

(C) \( x = 2.4 \)

(D) \( x = -2 \)

**Ans. (C)**

**Sol.**

Given: The mathematical model of control system

\[
\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 12(1 - e^{-2t})
\]

Where \( x(t) \) is the response of the system.

Laplace transform with initial condition assumed to be zero.

\( x(0) = 0 \) and \( x'(0) = 0 \)
\[
\begin{align*}
\left[ s^2X(s) - s x(0) - x'(0) \right] + 6 \left[ s X(s) - x(0) \right] + 5 X(s) &= 12 \left[ \frac{1}{s} - \frac{1}{s+2} \right] \\
(s^2 + 6s + 5)X(s) &= 12 \frac{2}{s(s+2)} \\
X(s) &= \frac{24}{s(s+2)(s^2 + 6s + 5)}
\end{align*}
\]

Applying final value theorem,
\[
\lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s) = \lim_{s \to 0} \frac{24}{(s+2)(s^2 + 6s + 5)}
\]
\[
x(\infty) = 2.4
\]

Hence, the correct option is (C).

**Question 5 [Work Book]**

In the feedback scheme shown in figure, the time-constant of the closed system will be

- (A) \( A \beta \tau \)
- (B) \( (1 + A \beta \tau) \)
- (C) \( \tau \)
- (D) \( \frac{\tau}{(1 + A \beta)} \)

**Ans.** (D)

**Sol.** Given block diagram is shown below.

Transfer function of system is given by,
\[
\begin{align*}
\frac{C(s)}{R(s)} &= \frac{A}{1 + \frac{A \beta}{1 + s \tau}} = \frac{A}{1 + A \beta + s \tau} \\
\frac{C(s)}{R(s)} &= \frac{A}{1 + \frac{s \tau}{1 + A \beta}} = \frac{A'}{1 + s \tau'}
\end{align*}
\]

where \( A' = \frac{A}{1 + A \beta} \) = closed loop dc gain of system

\( \tau' = \frac{\tau}{1 + A \beta} \) = closed loop time constant of system

Hence, the correct option is (D).

**Question 19 [Practice Book]**

A system described by the following differential equation \( \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y = x(t) \) is initially at rest.

For input \( x(t) = 2u(t) \), the output \( y(t) \) is

- (A) \( (1 - 2e^{-t} + e^{-2t})u(t) \)
- (B) \( (1 + 2e^{-t} - 2e^{-2t})u(t) \)
- (C) \( (0.5 + e^{-t} + 1.5e^{-2t})u(t) \)
- (D) \( (0.5 + 2e^{-t} + 2e^{-2t})u(t) \)
Given: \[ \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 2y = x(t) \text{ and } x(t) = 2u(t) \]

Where \( x(t) \) is the input and \( y(t) \) is the output.

Taking Laplace transform,

\[ X(s) = \frac{2}{s} \]

Using Laplace transform with zero initial conditions.

\[ s^2 Y(s) + 3sY(s) + 2Y(s) = X(s) \]

\[ (s^2 + 3s + 2)Y(s) = \frac{2}{s} \]

\[ Y(s) = \frac{2}{s(s + 2)(s + 1)} \]

By applying partial fraction, we get

\[ \frac{2}{s(s + 2)(s + 1)} = \frac{A}{s} + \frac{B}{s + 2} + \frac{C}{s + 1} \]

\[ A = 1, \ B = 1 \text{ and } C = -2 \]

Hence,

\[ Y(s) = \frac{1}{s} + \frac{1}{s + 2} - \frac{2}{s + 1} \]

Taking inverse Laplace transform, we get

\[ y(t) = [1 + e^{-2t} - 2e^{-t}]u(t) \]

Hence, the correct option is (A).

Consider the following systems

System 1: \( G(s) = \frac{1}{(2s + 1)} \), System 2: \( G(s) = \frac{1}{(5s + 1)} \)

The true statement regarding the system is

(A) Bandwidth of system 1 is greater than the bandwidth of system 2.
(B) Bandwidth of system 1 is lower than the bandwidth of system 2.
(C) Bandwidth of both the systems are the same.
(D) Bandwidth of both system are infinite.

Ans. (A)

Sol. Given: System 1: \( G(s) = \frac{1}{(2s + 1)} \), System 2: \( G(s) = \frac{1}{(5s + 1)} \)

For first order system, bandwidth is reciprocal of time constant.

Standard representation of first order system is given by,

\[ G(s) = \frac{1}{\tau s + 1} \text{ where } \tau = \text{time constant} \]

\[ G(s) = \frac{1}{(2s + 1)}, \quad \tau = 2 \text{ sec} \]

\[ G(s) = \frac{1}{(5s + 1)}, \quad \tau = 5 \text{ sec} \]

Bandwidth (system 1) = \( \frac{1}{\tau} = \frac{1}{2} \)

Bandwidth (system 2) = \( \frac{1}{\tau} = \frac{1}{5} \)

Bandwidth (system 1) is greater than bandwidth (system 2).

Hence, the correct option is (A).
Question 21 [Practice Book]  [GATE EC 2005 IIT-Bombay : 1 Mark]

Despite the presence of negative feedback, control systems still have problems of instability because the
(A) Components used have non-linearity.
(B) Dynamic equations of the subsystems are not known exactly.
(C) Mathematical analysis involves approximations.
(D) System has large negative phase angle at high frequencies.

Ans. (A)
Sol. Despite the presence of negative feedback, control systems still have problems of instability because the components used have non-linearity which introduces instability in the system.
Hence, the correct option is (A).

Question 23 [Practice Book]  [GATE IN 2007 IIT-Kanpur : 1 Mark]

A feedback control system with high gain \( K \), is shown in the figure below.

Then the closed loop transfer function is
(A) Sensitive to perturbations in \( G(s) \) and \( H(s) \).
(B) Sensitive to perturbations in \( G(s) \) but not to perturbations in \( H(s) \).
(C) Sensitive to perturbations in \( H(s) \) but not to perturbations in \( G(s) \).
(D) Insensitive to perturbations in \( G(s) \) and \( H(s) \).

Ans. (C)
Sol. The given figure is shown below.

Transfer function is given by,
\[
C = \frac{KG}{R + KGH}
\]
\[
C = \frac{KG}{1 + KGH} \times R
\]

If \( K \) is very high then \( 1 + KGH \approx KGH \)
\[
C = \frac{KG}{1 + KGH} \times R \approx \frac{KG}{KGH} \times R \approx \frac{1}{H} \times R
\]

It is clear from the above equation that when gain \( K \) is very high output of system is not affected by \( G \) but it is inversely proportional to feedback path gain \( H \).
Hence, the correct option is (C).

Question 26 [Practice Book]  [GATE EE 2009 IIT-Roorkee : 1 Mark]

The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as \( G_1 \), \( G_2 \) and \( \frac{1}{G_3} \). The relative small errors associated with each respective subsystem \( G_1 \), \( G_2 \) and \( G_3 \) are \( \varepsilon_1 \), \( \varepsilon_2 \) and \( \varepsilon_3 \). The error associated with the output is

(A) \( \varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3} \)
(B) \( \frac{\varepsilon_1 \varepsilon_2}{\varepsilon_3} \)
(C) \( \varepsilon_1 + \varepsilon_2 - \varepsilon_3 \)
(D) \( \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \)
Ans. **(C)**

Sol. **Given:**

Overall transfer function,

\[ G(s) = \frac{C(s)}{R(s)} = G_1 G_2 \times \frac{1}{G_3} \]

Output

\[ C = \frac{G_1 G_2}{G_3} \times R \]

\[ \ln C = \ln G_1 + \ln G_2 - \ln G_3 + \ln R \]

Differentiating,

\[ \frac{1}{C} \frac{dC}{ds} = \frac{1}{G_1} \frac{dG_1}{ds} + \frac{1}{G_2} \frac{dG_2}{ds} - \frac{1}{G_3} \frac{dG_3}{ds} \]

As no error in \( R \) so \( dR = 0 \)

\[ \frac{dC}{C} = \frac{1}{G_1} \frac{dG_1}{G_1} + \frac{1}{G_2} \frac{dG_2}{G_3} - \frac{1}{G_3} \frac{dG_3}{G_3} \]

\[ \varepsilon = \varepsilon_1 + \varepsilon_2 - \varepsilon_3 \]

So relative error in output \( \varepsilon = \varepsilon_1 + \varepsilon_2 - \varepsilon_3 \)

Hence, the correct option is (C).

---

**Question 27 [Practice Book]**

The response \( h(t) \) of a linear time invariant system to an impulse \( \delta(t) \), under initially relaxed condition is \( h(t) = e^{-t} + e^{-2t} \). The response of this system for a unit step input \( u(t) \) is

(A) \( u(t) + e^{-t} + e^{-2t} \)

(B) \( (e^{-t} + e^{-2t})u(t) \)

(C) \( (1.5 - e^{-t} - 0.5 e^{-2t})u(t) \)

(D) \( e^{-t}\delta(t) + e^{-2t}u(t) \)

Ans. **(C)**

Sol. **Given:** Impulse response \( h(t) = e^{-t} + e^{-2t} \)

Transfer function of a system is Laplace transform of its impulse response.

Transfer function = \( L(\text{Impulse response}) \)

\[ H(s) = L(e^{-t} + e^{-2t}) \]

\[ H(s) = \frac{1}{s+1} + \frac{1}{s+2} \]

\[ H(s) = \frac{C(s)}{R(s)} = \frac{1}{s+1} + \frac{1}{s+2} \]

\[ R(s) = \frac{1}{s} \text{ (step input)} \]

\[ C(s) = R(s) \cdot H(s) = \frac{1}{s} \left( \frac{1}{s+1} + \frac{1}{s+2} \right) \]

\[ C(s) = \frac{1}{s(s+1)} + \frac{1}{s(s+2)} \]

\[ C(s) = \left( \frac{1}{s} - \frac{1}{s+1} \right) + \frac{1}{2} \left( \frac{1}{s} - \frac{1}{s+2} \right) \]

\[ C(s) = \frac{1.5}{s+1} - \frac{0.5}{s+2} \]}
Step Response:

\[ c(t) = L^{-1} \left[ \frac{1.5}{s} - \frac{1}{s+1} - \frac{0.5}{s+2} \right] \]

\[ c(t) = (1.5 - e^{-t} - 0.5 e^{-2t}) u(t) \]

Hence, the correct option is (C).

**Question 1 [Work Book]**

The input \(-3e^{2t} u(t)\), where \(u(t)\) is the unit step function, is applied to a system with transfer function \(\frac{s-2}{s+3}\). If the initial value of the output is \(-2\), then the value of the output at steady state is _________.

**Ans.** 0

**Sol.** Given: \( r(t) = -3e^{2t} u(t) \), \( H(s) = \frac{Y(s)}{R(s)} = \frac{s-2}{s+3} \) and initial value of the output is \(-2\).

\[ r(t) = -3e^{2t} u(t) \]

Taking Laplace transform of \( r(t) \), we get \( R(s) = \frac{-3}{s-2} \)

Using final value theorem steady state output can be written as,

\[ y_{ss} = \lim_{s \to 0} \left( \frac{s-2}{s+3} \right) \left( \frac{-3}{s-2} \right) = \lim_{s \to 0} \left( \frac{-3s}{s+3} \right) = 0 \]

Hence, the answer is 0.

**Question 8 [Work Book]**

A plant has an open-loop transfer function,

\[ G_p(s) = \frac{20}{(s + 0.1)(s + 2)(s + 100)} \]

The approximate model obtained by retaining only one of the above poles, which is closest to the frequency response of the original transfer function at low frequency is

\((A) \frac{0.1}{s + 0.1} \quad (B) \frac{2}{s+2} \quad (C) \frac{100}{s + 100} \quad (D) \frac{20}{s + 0.1} \)

**Ans.** (A)

**Sol.** Given: Open-loop transfer function

\[ G_p(s) = \frac{20}{(s + 0.1)(s + 2)(s + 100)} \]

At low frequency \(\omega \to 0\) and considering dominant pole concept the above transfer function can be written as,

\[ G_p(s) = \lim_{s \to 0} \frac{20}{(s + 0.1)(s + 2)(s + 100)} \]

\[ G_p(s) = \lim_{s \to 0} \frac{20}{(s + 0.1)(2)(100)} = \frac{0.1}{(s + 0.1)} \]
**Dominant pole**: The pole which dominates the step response of a system. These are the poles which are closest to the $j\omega$ axis. Dominant pole may be in the right half of $s$-plane.

Hence, the correct option is (A).

**Question 9 [Work Book]**

The unit step response of a system with the transfer function $G(s) = \frac{1-2s}{1+s}$ is given by which one of the following waveforms?

(A) ![Graph A](image1.png)

(B) ![Graph B](image2.png)

(C) ![Graph C](image3.png)

(D) ![Graph D](image4.png)

**Ans.** (A)

**Sol.** Given: $G(s) = \frac{1-2s}{1+s}$

The transfer function is:

$u(t) \rightarrow U(s) = \frac{1}{s}$

$Y(s) = G(s)U(s)$

$Y(s) = \frac{(1-2s)}{1+s} \cdot \frac{1}{s}$

$Y(s) = \frac{A}{s} + \frac{B}{s+1}$

$A = 1, B = -3$

$Y(s) = \frac{1}{s} + \frac{-3}{s+1}$
Taking inverse Laplace transform, we get
\[ y(t) = u(t) - 3e^{-t}u(t) \]
\[ y(t) = (1 - 3e^{-t})u(t) \]

Hence, the correct option is (A).

**Question 10 [Work Book]**

A first-order low-pass filter of time constant \( T \) is excited with different input signals (with zero initial conditions up to \( t = 0 \)). Match the excitation signals \( X, Y, Z \) with the corresponding time responses for \( t \geq 0 \)

- \( X \): Impulse
- \( Y \): Unit step
- \( Z \): Ramp

\[ P : 1 - e^{-t/T} \]
\[ Q : t - T(1 - e^{-t/T}) \]
\[ R : e^{-t/T} \]

(A) \( X \rightarrow R, Y \rightarrow Q, Z \rightarrow P \)
(B) \( X \rightarrow Q, Y \rightarrow P, Z \rightarrow R \)
(C) \( X \rightarrow R, Y \rightarrow P, Z \rightarrow Q \)
(D) \( X \rightarrow P, Y \rightarrow R, Z \rightarrow Q \)

**Ans.** (C)

**Sol.**

For first order system
\[ G(s) = \frac{1}{sT}; \quad H(s) = 1 \]

\[ \begin{array}{c}
R(s) \\
\hline
Y(s)
\end{array} \]

Closed loop transfer function is given by,
\[ \frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \]
\[ Y(s) = \frac{G(s)}{1 + G(s)H(s)} \cdot R(s) \]

For impulse response \( R(s) = 1 \)
\[ Y(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1/sT}{1 + 1/sT} \]
\[ Y(s) = \frac{1}{1 + sT} \]
\[ y(t) = \frac{1}{T} e^{-t/T} \text{ for } t \geq 0 \]

For step response \( R(s) = \frac{1}{s} \)
\[ Y(s) = \frac{1}{s(1 + sT)} = \frac{(1 + sT) - (sT)}{s(1 + sT)} \]
\[ Y(s) = \frac{1}{s} - \frac{T}{(1 + sT)} = \frac{1}{s} - \frac{T}{T(s + \frac{1}{T})} \]
For ramp response \( R(s) = \frac{1}{s^2} \)

\[
Y(s) = \frac{1}{s^2(1+sT)} = \frac{1}{s^2} \left( \frac{T}{s} + \frac{1}{1+\frac{T}{s}} \right)
\]

\[y(t) = t - T(1 - e^{-t/T}) \quad \text{for} \quad t \geq 0\]

Hence, the correct option is (C).

**Trick:**
You can directly check by
\[
\frac{d}{dt} \text{ (ramp response) = step response} \\
\frac{d}{dt} \text{ (step response) = impulse response}
\]

**Question 3 [Work Book]**  
**[GATE EC 2016 IISc-Bangalore (Set-2) : 1 Mark]**

The response of the system \( G(s) = \frac{s - 2}{(s + 1)(s + 3)} \) to the unit step input \( u(t) \) is \( y(t) \).

The value of \( \frac{dy}{dt} \) at \( t = 0^+ \) is \( \ldots \ldots \).

**Ans.** 1

**Sol.**  
Given: \( G(s) = \frac{(s - 2)}{(s + 1)(s + 3)} \) and \( r(t) = u(t) \)

Hence, \( R(s) = \frac{1}{s} \)

Laplace transform of the output is given by,
\[
Y(s) = G(s) \cdot R(s) \\
Y(s) = \frac{s - 2}{s(s + 1)(s + 3)}
\]

\( y(0) = \) initial value

Hence apply initial value theorem
\[
y(0) = \lim_{s \to \infty} sY(s)
\]

\[
y(0) = \lim_{s \to \infty} \frac{s - 2}{(s + 1)(s + 3)} = \left( \frac{1 - \frac{2}{s}}{s} \right) \left( \frac{1 + \frac{1}{s}}{1 + \frac{3}{s}} \right)
\]

\( y(0) = 0 \)

\[
L \left[ \frac{dy}{dt} \right] = sY(s) - y(0)
\]

\[
L \left[ \frac{dy}{dt} \right] = sY(s) = \frac{s(s - 2)}{s(s + 1)(s + 3)} = \frac{(s - 2)}{(s + 1)(s + 3)}
\]

\[
\left. \frac{dy}{dt} \right|_{t=0} = \lim_{s \to \infty} sL \left[ \frac{dy}{dt} \right]
\]

\[
\left. \frac{dy}{dt} \right|_{t=0} = \lim_{s \to \infty} \frac{s(s - 2)}{s(s + 1)(s + 3)} = \left( \frac{1 - \frac{2}{s}}{1 + \frac{1}{s}} \right) \left( \frac{1}{1 + \frac{3}{s}} \right) = 1
\]

Hence, the correct answer is 1.
The relationship between the force \( f(t) \) and the displacement \( x(t) \) of a spring-mass system (with mass \( M \), viscous damping \( D \) and spring constant \( K \)) is

\[
M \frac{d^2x(t)}{dt^2} + D \frac{dx(t)}{dt} + Kx(t) = f(t)
\]

\( X(s) \) and \( F(s) \) are the Laplace transforms of \( x(t) \) and \( f(t) \) respectively. With \( M = 0.1, D = 2, K = 10 \) in appropriate units, the transfer function \( G(s) = \frac{X(s)}{F(s)} \) is

\[
\text{(A)} \quad \frac{10}{s^2 + 20s + 100} \quad \text{(B)} \quad s^2 + 20s + 100 \quad \text{(C)} \quad \frac{10s^2}{s^2 + 20s + 100} \quad \text{(D)} \quad \frac{s}{s^2 + 20s + 100}
\]

Ans. (A)

Sol.

\[
M \frac{d^2x(t)}{dt^2} + D \frac{dx(t)}{dt} + Kx(t) = f(t)
\]

\[
X(S) = \frac{1}{F(S)} \quad MS^2 + DS + K
\]

\[
\text{(A)} \quad \frac{2(s + 1)}{s^2 + 4s + 5} \quad \text{(B)} \quad \frac{5(s + 1)}{s^2 + 4s + 5} \quad \text{(C)} \quad \frac{10(s + 1)}{s^2 + 4s + 5} \quad \text{(D)} \quad \frac{10(s + 1)}{(s + 2)^2}
\]

Ans. (C)

Sol. Given: Steady state gain = 2

Given pole zero plot is shown in figure below.
Poles are \( s = -2 \pm j \) and zero at \( s = -1 \).

Transfer function can be written as,

\[
T(s) = \frac{K(s+1)}{(s+2+j)(s+2-j)(s+4s+5)} = \frac{K(s+1)}{(s^2 + 4s + 5)}
\]

Hence, the correct option is (C).

**Question 15 [Practice Book] [ESE EE 1997]**

If the unit step response of a network is \( (1 - e^{-\alpha t}) \), then its unit impulse response will be

(A) \( \alpha e^{-\alpha t} \)  
(B) \( \alpha e^{\alpha t} \)  
(C) \( \frac{1}{\alpha} e^{-\alpha t} \)  
(D) \( (1 - \alpha)e^{-\alpha t} \)

Ans. (A)

Sol. Given: The unit step response of network is

\( c(t) = 1 - e^{-\alpha t} \)

\[
\frac{d}{dt} (\text{Unit step response}) = \text{Impulse response}
\]

\[
\frac{d}{dt} (1 - e^{-\alpha t}) = \alpha e^{-\alpha t}
\]

Hence, the correct option is (A).

**Question 24 [Practice Book] [ESE EC 2002]**

Consider the following single-loop feedback structure illustrating the return difference

The return difference for \( A \) is

(A) \( 1 - A \beta \)  
(B) \( 1 + A \beta \)  
(C) \( \frac{A \beta}{1 + A \beta} \)  
(D) \( \frac{A \beta}{1 - A \beta} \)

Ans. (B)

Sol. Overall transfer function \( M(s) = \frac{G(s)}{1 + G(s)H(s)} \)

Sensitivity by with respect to \( G(s) \)

\[
S_d^{\text{ut}} = \frac{\partial M}{\partial A} = \frac{\partial M}{\partial A} \frac{A}{M} = \frac{1}{1 + A \beta}
\]

The return difference for \( A \) is called desensitivity.

Desensitivity \( = \frac{1}{\text{Sensitivity}} = 1 + A \beta \)

Hence, the correct option is (B).
A linear network has the system function

\[ H(s + c) \]

\[ \frac{(s + a)(s + b)}{(s + a)(s + b)} \]

The outputs of the network with zero initial conditions for two different inputs are tabulated as

<table>
<thead>
<tr>
<th>Input ( x(t) )</th>
<th>Output ( y(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u(t) )</td>
<td>( 2 + De^{-t} + Ee^{-3t} )</td>
</tr>
<tr>
<td>( e^{-2t}u(t) )</td>
<td>( Fe^{-t} + Ge^{-3t} )</td>
</tr>
</tbody>
</table>

Then the values of \( c \) and \( H \) are, respectively

(A) 2 and 3  
(B) 3 and 2  
(C) 2 and 2  
(D) 1 and 3

**Ans.** (A)

**Sol.**

Given:

\[ T(s) = H \frac{(s + c)}{(s + a)(s + b)} \]  
\[ \ldots (i) \]

When input is \( u(t) \) output is,

\[ 2 + De^{-t} + Ee^{-3t} \]

When input is \( e^{-2t}u(t) \) output is,

\[ Fe^{-t} + Ge^{-3t} \]

Using equation (i), when input is \( u(t) \) output is,

\[ H \frac{(s + c)}{s(s + a)(s + b)} = \frac{K}{s} + \frac{D}{s + a} + \frac{E}{s + b} \]

Taking inverse Laplace transform, we get

\[ 2 + De^{-t} + Ee^{-3t} \]

So, \( a = 1 \) and \( b = 3 \)

Using final value theorem,

\[ \lim_{t \to \infty} \frac{s \cdot H(s + c)}{s(s + a)(s + b)} = \lim_{t \to \infty} 2 + De^{-t} + Ee^{-3t} \]

\[ \frac{Hc}{ab} = 2 \text{ and } Hc = 6 \]

Using equation (i) when input is \( e^{-2t}u(t) \) output is,

\[ H \frac{(s + c)}{(s + 2)(s + a)(s + b)} \]

Only two terms are present in the response.

Hence \( \frac{s + c = s + 2}{c = 2} \)

\[ H = 3 \quad (\because \ Hc = 6) \]

Hence, the correct option is (A).

---

**Question 34 [Practice Book] [ESE EC 2007]**

For the system given below, the feedback does not reduce the closed-loop sensitivity due to variation of which one of the following?

(A) \( K \)  
(B) \( A \)  
(C) \( K\alpha \)  
(D) \( \beta \)
Ans. (C)
Sol. Given system is shown below

\[ G(s) = \frac{KA}{s + \alpha} \quad \text{… Transfer function without feedback} \]

\[ F(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{KA/(s + \alpha)}{1 + KA/(s + \alpha)} \cdot \beta = \frac{KA}{s + \alpha + K\beta} \quad \text{… T.F. with Feedback} \]

Sensitivity of \( G(s) \) with respect to ‘\( A \)’

\[ S_A^{G(s)} = \frac{A}{G(s)} \frac{\partial G(s)}{\partial A} = \frac{A}{KA/(s + \alpha)} \cdot \frac{(s + \alpha) \cdot K - KA \cdot 0}{(s + \alpha)^2} \]

\[ = \frac{(s + \alpha)(s + \alpha)K}{K} \cdot \frac{(s + \alpha + K\beta)}{(s + \alpha)^2} = 1 \]

Sensitivity of \( F(s) \) with respect to ‘\( A \)’

\[ S_A^{F(s)} = \frac{A}{F(s)} \frac{\partial F(s)}{\partial A} = \frac{A}{KA/(s + \alpha + K\beta)} \cdot \frac{(s + \alpha + K\beta) \cdot K - KA(K\beta)}{(s + \alpha + K\beta)^2} \]

\[ = \frac{(s + \alpha + K\beta) \cdot K(s + \alpha + K\beta - K\beta)}{(s + \alpha + K\beta)^2} \]

\[ S_A^{F(s)} = \frac{s + \alpha}{s + \alpha + K\beta} \]

\[ \therefore \quad S_A^{G(s)} \neq S_A^{F(s)} \quad \text{i.e. sensitivity is reduced due to feedback} \]

Similarly \( S_K^{G(s)} \neq S_K^{F(s)} \)

\[ S_K^{G(s)} = 1 \quad S_K^{F(s)} = \frac{s + \alpha}{s + \alpha + K\beta A} \]

Sensitivity of \( G(s) \) with respect to \( \beta \)

\[ S_{\beta}^{G(s)} = \frac{\beta}{G(s)} \frac{\partial G(s)}{\partial \beta} = \frac{\beta}{KA/(s + \alpha)} \cdot \frac{(s + \alpha) \cdot 0 - KA \cdot 0}{(s + \alpha)^2} \]

\[ \therefore \quad S_{\beta}^{G(s)} = 0 \]

Sensitivity of \( F(s) \) with respect to \( \beta \)

\[ S_{\beta}^{F(s)} = \frac{\beta}{F(s)} \frac{\partial F(s)}{\partial \beta} = \frac{\beta}{KA/(s + \alpha + K\beta A)} \cdot \frac{(s + \alpha + K\beta A) \cdot 0 - KA(KA)}{(s + \alpha + K\beta A)^2} \]

\[ = \frac{\beta \cdot (s + \alpha + K\beta A)(-K^2 A^2)}{KA(s + \alpha + K\beta A)^2} \]

\[ = \frac{-K\beta}{(s + \alpha + K\beta A)} \]

Hence, the sensitivity is reduced due to feedback.
Sensitivity of $G(s)$ with respect to $K\alpha$

$$S_{Ku}^{G(s)} = \frac{K\alpha}{G(s)} \cdot \frac{\partial G(s)}{\partial (K\alpha)} = \frac{K\alpha}{KA / s + \alpha} \cdot \frac{\partial}{\partial (K\alpha)} \left( \frac{KA}{s + \alpha} \right)$$

$$S_{Ku}^{G(s)} = \frac{\alpha(s + \alpha)}{A} \cdot \frac{(s + \alpha)0 - KA\cdot0}{(s + \alpha)^2}$$

$$\therefore S_{Ku}^{G(s)} = 0$$

Sensitivity of $F(s)$ with respect to $K\alpha$

$$S_{Ku}^{F(s)} = \frac{K\alpha}{F(s)} \cdot \frac{\partial F(s)}{\partial (K\alpha)} = \frac{K\alpha}{KA / (s + \alpha + K\alpha\beta)} \cdot \frac{\partial}{\partial (K\alpha)} \left( \frac{KA}{s + \alpha + K\alpha\beta} \right)$$

$$S_{Ku}^{F(s)} = \frac{\alpha(s + \alpha + K\alpha\beta)}{A} \cdot \frac{(s + \alpha + K\alpha\beta)0 - KA\cdot0}{(s + \alpha + K\alpha\beta)^2}$$

$$\therefore S_{Ku}^{F(s)} = 0$$

Hence, $S_{Ku}^{G(s)} = S_{Ku}^{F(s)}$

So, the sensitivity with respect to $K\alpha$ does not reduce due to feedback.

Hence, the correct option is (C).

**Question 12 [Work Book] [ESE EC 2012]**

The sensitivity $S_K^T$ of transfer function $T = \frac{(1+2K)}{(3+4K)}$ with respect to the parameter $K$ is given by

(A) $\frac{K}{3+K^2}$

(B) $\frac{3K}{2+4K+K^2}$

(C) $\frac{2K}{3+10K+8K^2}$

(D) $\frac{4K}{2+5K-7K^2}$

**Ans.** (C)

**Sol.** Given: $T = \frac{(1+2K)}{(3+4K)}$

Sensitivity of 'T' with respect to changes in 'K' is $S_K^T$ and can be written as,

$$S_K^T = \frac{\% \text{change in } T}{\% \text{ change in } K} = \frac{\frac{\partial T}{T}}{\frac{\partial K}{K}} = \frac{\partial T}{\partial K} \cdot \frac{K}{T}$$

$$S_K^T = \frac{\partial}{\partial K} \left( \frac{1+2K}{3+4K} \right) \frac{K}{(1+2K)} \left(3+4K\right)$$

$$S_K^T = \frac{(3+4K)^2 - (1+2K)4}{(3+4K)^2} \frac{K(3+4K)}{(1+2K)}$$

$$S_K^T = \frac{(6+8K-4-8K)K}{(3+4K)(1+2K)} = \frac{2K}{8K^2+10K+3}$$

Hence, the correct option is (C).

**Question 51 [Practice Book] [ESE EE 2013]**

Consider the following statements regarding advantages of closed loop negative feedback control systems over open loop systems.

1. The overall reliability of the closed loop system is more than that of open loop system.
2. The transient response in a closed loop system decays more quickly than in open loop system.
3. In an open loop system, closing of the loop increases the overall gain of the system.
4. In the closed loop system, the effect of variation of component parameters on its performance is reduced.
Which of these statements are correct?
(A) 1 and 2  
(B) 1 and 3  
(C) 2 and 4  
(D) 3 and 4  

Ans. (A)  
Sol. (i) For negative feedback control system, overall gain reduces by factor \((1 + GH)\). So option (B) and (D) cannot be correct.
(ii) The transient response in a closed loop system decay more quickly because time constant for closed-loop system is less than open loop system.
(iii) Overall reliability of closed-loop system is more than open loop system. Hence, the correct option is (A).

Question 54 [IES EE 2013]

A forcing function \((t^2 - 2t)u(t-1)\) is applied to a linear system. The Laplace transform of the forcing function is

\[ (A) \frac{2-s}{s^3}e^{-2s} \quad (B) \left(\frac{1-s^2}{s}\right)e^{-s} \quad (C) \frac{1}{s}e^{-s} - \frac{1}{s^2}e^{-2s} \quad (D) \left(\frac{2-s^2}{s^3}\right)e^{-s} \]

Ans. (D)  
Sol. Given: \[ f(t) = (t^2 - 2t)u(t-1) = [(t-1)^2 - 1]u(t-1) \]
\[ f(t) = (t-1)^2 u(t-1) - u(t-1) \]
Taking Laplace transform, we get
\[ F(s) = \frac{2e^{-s}}{s^3} - \frac{e^{-s}}{s} = \left(\frac{2-s^2}{s^3}\right)e^{-s} \]

Hence, the correct option is (D).

Question 56 [Practice Book] [ESE EE 2014]

The unit impulse response of a system given as \(c(t) = -4e^{-t} + 6e^{-2t}\). The step response of the same system for \(t \geq 0\) equal to

\(A) 3e^{-2t} + 4e^{-t} + 1 \quad (B) -3e^{-2t} + 4e^{-t} + 1 \quad (C) -3e^{-2t} + 4e^{-t} - 1 \quad (D) 3e^{-2t} - 4e^{-t} + 1 \)

Ans. (C)  
Sol. Given: \(c(t) = -4e^{-t} + 6e^{-2t}\)
Step response can be calculated as,
\[ \text{Step response} = \int \text{impulse response} \]
\[ \text{Step response} = \int (-4e^{-t} + 6e^{-2t}) \, dt \]
\[ \text{Step response} = 4e^{-t} - 3e^{-2t} + C \]
At \(t = 0\), step response is zero.
\[ 0 = 4 - 3 + C \]
\[ C = -1 \]
\[ \text{Step response} = 4e^{-t} - 3e^{-2t} - 1 \]
Hence, the correct option is (C).